Closed-Loop Identification of a Velocity Controlled DC Servomotor

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Abstract — **This paper presents a methodology for closed-loop identification of velocity controlled servomotors. The approach considers a PI controller applied simultaneously to the real servomotor and its model. It is shown that under persistently exciting conditions the parameter estimates converges. Experimental results using a laboratory test bed validate the proposed approach.**

Keywords — **Parameter estimation, persistent excitation, DC servomechanism**

I. INTRODUCTION

Servomotors are key elements in a great variety of industrial applications requiring speed and position control. The knowledge of the servomotor parameters is necessary to improve their transient, steady state and dynamic characteristics. For this reason, parameter identification techniques are a first step prior to setting up a model-based control law. A linear model of a servomotor considering its angular velocity as the controlled variable is open loop stable; therefore, many of the identification techniques in the literature are adequate for obtaining parameter estimates. However, in some cases, it would be interesting to identify a system in closed loop; for instance, if the open loop gain is high, then, small input values would produce large output values that would not be desirable for security reasons. This situation occurs in servomotors working in velocity mode; if a current-controlled amplifier feeds the motor, the open loop gain is high. On the other hand, having a low open loop gain may produce a sluggish response and the identification procedure may slow down. A way to cope with the aforementioned problems is to modify the system gain using feedback. At this stage it is interesting to point out that velocity regulation of servomotors is accomplished in many industrial applications using Proportional Integral controllers [1].

There are several works dealing with parameter estimation of servomotors considering velocity as the output variable. Works in this vein are [2], [3], [4], [5], [6], [7], [8], [9], [10]. In [2] the authors propose a graphical method using velocity step responses. Frequency based methods are studied in [3], [4], [5], [6]. In [3] the authors employ an off-line Least Squares algorithm in the frequency domain, compare it against a standard time domain Least Squares algorithm and show that the former has better performance. Velocity and armature voltage feed the estimation algorithms. An interesting feature is the multifrequency excitation signal used to perform the parameter estimation, however, the authors underline that the frequency approach is not suitable for on-line DC motor parameter identification. Reference [4] is an interesting survey about estimation of nonlinear models of DC motors. In this case, using armature current measurements and assuming that the motor works in open loop, the authors show that it is possible to estimate the servomotor parameters through the so-called Hartley functions. These functions allow converting a nonlinear differential equation into an algebraic relationship in the same way that the Laplace transform is applied to a linear differential equation with constant parameters. The off-line frequency-weighted Least Squares algorithm estimates the unknown parameters. The approach described in [6] also employs the Least Squares method where the output variable is the motor velocity; all the experiments are performed in open loop. In [7] an off-line Maximum Likelihood algorithm permits servomotor identification considering the presence of nonlinear friction. A feature of the method is the use of multifrequencial binary excitation signals which overcome the identification errors introduced by the presence of Coulomb friction. The authors employ open loop experiments and the measured variables are both the servomotor angular velocity and the rectilinear velocity of a table driven by the servomotor through a ball screw. Reference [8] describes another interesting approach. In this case, the servomotor operates in open loop and a recursive on-line Least Squares algorithm with forgetting factor estimates the servo parameters using a Hammerstein model. Reference [9] proposes a similar approach. It is worth mentioning the approach in [10] where the authors apply closed loop identification techniques to a rotating three-mass electromechanical system. A recursive Least Squares algorithm with forgetting factor performs parameter identification employing a discrete-time model. The servomotor speed and the armature voltage feed the identification algorithm. A Proportional Integral (PI) velocity controller controls the servomotor for the closed loop experiments. Convergence of the parameter estimates is rather slow in open and closed loop; however, it is experimentally shown that convergence rates in closed loop are faster than in open loop. Algebraic tools are another approach for parameter identification [11]. In that reference, the authors apply their methodology by means of numerical simulations to several plants. The plant is firstly identified then controlled using an algorithm computed employing the

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parameter estimates previously estimated. From the review it is clear that most of the approaches rely on open loop experiments and that, the Least Squares algorithm is in most cases the main tool for parameter identification.

This work presents some preliminary results concerning an identification methodology for a velocity-controlled servomotor. A depart from most previous approaches is the fact that parameter identification is performed while the motor works in closed loop under PI control. Moreover, the analysis of the identification algorithm takes into account the PI controller. Section 2 presents the proposed identification methodology. Section 3 shows some experimental results that sustain the proposed approach. Finally, the paper ends with some concluding remarks.

II. IDENTIFICATION ALGORITHM

A. Preliminaries

The following paragraphs describe the proposed identification approach. Consider a servomotor driven by a current amplifier. Two identical PI controllers close the loop around the servomotor and a model as depicted in Fig. 01. The difference between the velocity of the servomotor and the estimated velocity generated by the model feeds the identifier; subsequently, the parameter estimates update the model. By using the PI controller, the servomotor has a stable behavior despite the amplifier gain. However, even when the closed-loop system associated to the servomotor is stable, it is not the case for the model; therefore, stability of the model needs further study.

The servomotor working in velocity mode together with a current amplifier obeys the following model

$$J\dot{\omega}(t) + f\omega(t) = ku(t) \tag{1}$$

where $\omega(t)$ represents the servomotor shaft angular velocity, J and f are the servomotor inertia and viscous friction, $u(t)$ is the amplifier input voltage and k stands for the amplifier gain. Rewriting (1) gives

$$\dot{\omega}(t) = -a\omega(t) + bu(t) \tag{2}$$

with $a=f/J$, $b=k/J$ being positive constants. Consider the PI control law

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau)d\tau \tag{3}$$

and the velocity error

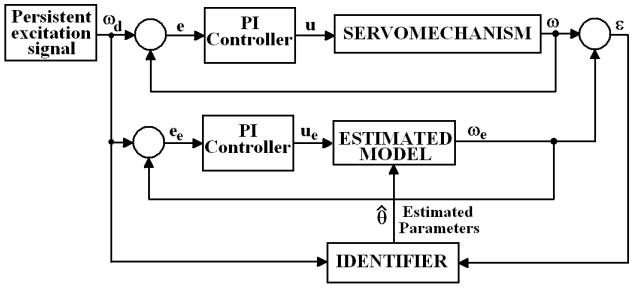

Fig. 1. Block diagram of the proposed identification method.

$$e(t) = \omega_d - \omega(t) \tag{4}$$

with ω_d a reference. Consider now the following estimated model

$$\dot{\omega}_e(t) = -\hat{a}\omega_e(t) + \hat{b}u_e(t) \tag{5}$$

where $\hat{a}$ and $\hat{b}$ being estimates of a and b respectively, in closed-loop with the PI controller

$$u_e(t) = k_p e_e(t) + k_i \int_0^t e_e(\tau)d\tau \tag{6}$$

The next expression defines the velocity error associated to the model

$$e_e(t) = \omega_d - \omega_e(t) \tag{7}$$

Note that the control laws (3) and (6) uses the same gains. Sustituting (3) into (2) and (6) into (5) yields

$$\dot{\omega}(t) = -a\omega(t) + bk_p e(t) + bk_i \int_0^t e(\tau)d\tau \tag{8}$$

$$\dot{\omega}_e(t) = -\hat{a}\omega_e(t) + \hat{b}k_p e_e(t) + \hat{b}k_i \int_0^t e_e(\tau)d\tau \tag{9}$$

Define the error between the outputs of the servomotor and its model as

$$\varepsilon(t) = \omega(t) - \omega_e(t) \tag{10}$$

Hence, employing (8) and (9), the time derivative of (10) is given by

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$$\begin{aligned}\dot{\varepsilon} &= \dot{\omega} - \dot{\omega}_e \\ &= -a\omega + bk_p e + bk_i \int_0^t e\, d\tau \\ &+ \hat{a}\omega_e - \hat{b}k_p e_e - \hat{b}k_i \int_0^t e_e d\tau\end{aligned}$$

Adding and substracting $a\omega_e$, $bk_p e_e$ and $bk_i \int_0^t e_e d\tau$ from the right-hand side of this last expression leads to

$$\begin{aligned}\dot{\varepsilon} &= -a\varepsilon - bk_p\varepsilon - bk_i \int \varepsilon\, d\tau \\ &+ (\hat{a} - a)\omega_e - (\hat{b} - b)\left[k_p e_e + k_i \int e_e d\tau\right]\end{aligned}$$

Define

$$\begin{aligned}z(t) &:= \int_0^t \varepsilon(\tau) d\tau \\ \alpha &:= a + bk_p \\ \tilde{\theta} &:= \hat{\theta} - \theta = \begin{bmatrix}\hat{a} - a \\ \hat{b} - b\end{bmatrix} \\ \phi(t) &:= \begin{bmatrix}\omega_e(t) \\ -k_p e_e(t) - k_i \int_0^t e_e(\tau) d\tau\end{bmatrix}\end{aligned} \tag{11}$$

then, $\dot{\varepsilon}$ becomes

$$\dot{\varepsilon} = -\alpha\varepsilon - bk_i z + \tilde{\theta}^T \phi(t) \tag{12}$$

B. Stability analysis

This section deals with the stability analysis of (12) which allows concluding stability of the model (9). To this end consider next Lyapunov candidate function

$$\begin{aligned}V(\varepsilon, z) &= \frac{1}{2}\begin{bmatrix}\varepsilon & z\end{bmatrix}\begin{bmatrix}1 & \mu \\ \mu & bk_i\end{bmatrix}\begin{bmatrix}\varepsilon \\ z\end{bmatrix} \\ &+ \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta}\end{aligned} \tag{13}$$

where $\Gamma = \Gamma^T \in \Re^{2x2}$ is positive definite and $\mu \in \Re$. It is clear that (13) is positive definite if the next inequality holds

$$\mu < \sqrt{bk_i} \tag{14}$$

Taking the time derivative of (13) along the trajectories of (12) yields

$$\begin{aligned}V(\varepsilon, z) &= \varepsilon\dot{\varepsilon} + \mu z\dot{\varepsilon} + \mu\dot{z}\varepsilon + bk_i z\dot{z} + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\ &= \varepsilon\left[-\alpha\varepsilon - bk_i z + \tilde{\theta}^T \phi\right] + \mu z\left[-\alpha\varepsilon - bk_i z + \tilde{\theta}^T \phi\right] \\ &+ \mu\varepsilon^2 + bk_i z\varepsilon + \tilde{\theta}^T \Gamma^{-1} \dot{\tilde{\theta}} \\ &= -(\alpha - \mu)\varepsilon^2 - \mu bk_i z^2 - \mu\alpha z\varepsilon \\ &+ \tilde{\theta}^T \left[\phi\varepsilon + \phi\mu z + \Gamma^{-1}\dot{\tilde{\theta}}\right]\end{aligned}$$

Since $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$, choosing the update law

$$\dot{\tilde{\theta}} = -\Gamma\phi(\mu z + \varepsilon) \tag{15}$$

and defining $\gamma = (\alpha - \mu) > 0$ allows writing $\dot{V}$ as

$$\begin{aligned}\dot{V}(\varepsilon, z) &= -\gamma\varepsilon^2 - \mu bk_i z^2 - \mu\alpha z\varepsilon \\ &\le -\left[\gamma - \frac{\mu\alpha^2}{4bk_i}\right]\varepsilon^2\end{aligned} \tag{16}$$

If μ fulfills the following inequality

$$\mu < \frac{4\alpha bk_i}{4bk_i + \alpha^2} \tag{17}$$

then (16) is negative semidefinite. Consequently, ε and $\tilde{\theta}$ are bounded. Define $\beta = \gamma - \mu\alpha^2/(4bk_i)$, thus, from (16) it follows that

$$\int_0^t \varepsilon^2(\tau) d\tau \le \frac{V(0) - V(t)}{\beta} < \infty \tag{18}$$

so $\varepsilon \in L_2 \cap L_\infty$ and $(d\varepsilon/dt) \in L_\infty$. Therefore, from Barbalat's lemma [12], it follows that $\varepsilon(t) \to 0$ as $t \to \infty$. Hence, $\omega_e(t) \in L_\infty$ and stability of (9) follows. Note, however, that the above analysis does not allows concluding convergence of $\tilde{\theta}$ to zero.

C. Parameter convergence

Using (12) and (15) and defining $y(t) := \mu z + \varepsilon$, it is possible to obtain the following state-space representation

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned} \tag{19}$$

$$\dot{\tilde{\theta}} = -\Gamma\phi y \tag{20}$$

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where

$$A = \begin{bmatrix} 0 & 1 \\ -bk_i & -\alpha \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, u = \tilde{\theta}^T \phi \qquad (21)$$
$$C = [\mu \quad 1], x(t) = \begin{bmatrix} z \\ \varepsilon \end{bmatrix}$$

The corresponding transfer function for (19) is given by

$$G(s) = C(sI - A)^{-1} B = \frac{s + \mu}{s^2 + \alpha s + bk_i} \qquad (22)$$

It is not difficult to obtain an expression for $G(s)$ evaluated in $j\omega$ in terms of its real and imaginary parts

$$G(s) = \mathrm{Re}[G(j\omega)] + j\,\mathrm{Im}[G(j\omega)]$$
$$\mathrm{Re}[G(j\omega)] = \frac{bk_i\mu + \omega^2(\alpha - \mu)}{(bk_i - \omega^2)^2 + (\alpha\omega)^2}$$
$$\mathrm{Im}[G(j\omega)] = \frac{(bk_i - \omega^2)\omega - \mu\alpha\omega}{(bk_i - \omega^2)^2 + (\alpha\omega)^2}$$

Note that

$$\lim_{\omega\to\infty} \omega^2 \mathrm{Re}[G(j\omega)] = \lim_{\omega\to\infty} \frac{bk_i\mu\omega^2 + \omega^4(\alpha - \mu)}{(bk_i - \omega^2)^2 + (\alpha\omega)^2}$$
$$= \lim_{\omega\to\infty} \frac{(\alpha - \mu) + \dfrac{bk_i\mu\omega^2}{\omega^4}}{1 + \dfrac{\alpha^2 - 2bk_i}{\omega^2} + \dfrac{b^2k_i^2}{\omega^4}} = \alpha - \mu$$

Then, according to standard results about Strictly Positive Real (SPR) functions, transfer function (22) is SPR if

$$\mu < \alpha \qquad (23)$$

The following definition about a Persistently Exciting (PE) signal [12] will be useful for stating parameter convergence.

Definition 1. A vector $\phi: \Re_+ \to \Re_{2n}$ is PE if there exist positive constants α_1, α_2, δ such that

$$\alpha_1 I \le \int_{t_0}^{t_0+\delta} \phi(\tau)\phi^T(\tau)d\tau \le \alpha_2 I, \forall t_0 \ge 0 \qquad (24)$$

Considering the updating equation (20) and using standard results on parameter convergence of adaptive identifiers using SPR equations [12], if the regressor vector $\phi(t)$ is PE, then $\tilde{\theta}$ converges exponentially to zero. It is worth to note that the signals in $\phi(t)$ belong to a time-varying system, namely, the estimated system (9). However, from the stability analysis performed in Subsection II-B, it follows that $\phi(t)$ corresponding to the estimated model converges to a vector $\phi_r(t)$ corresponding to the servomotor. Then, if $\phi_r(t)$ fulfills the PE condition so does $\phi(t)$. This observation allows concluding that it is only neccessary to establish the relationship between Sufficiently Richness (SR) of the reference signal ω_d and the PE conditions on ϕ_r, corresponding to the input and output of a linear time-invariant system respectively. Now, consider the following definition about SR signals [12].

Definition 2. A stationary signal $r(t): \Re_+ \to \Re^n$ is called SR of order n if the support of the spectral density of r(t), namely, $S_r(d\omega)$, contains at least n points.

The following Proposition will allow obtaining the main result of this subsection.

Proposition 1. Let $\phi \in \Re^n$ the output of a stable linear time-invariant system with transfer function L(s) and an input r(t) SR of order n. Assume that $L(j\omega_1)$, ..., $L(j\omega_n)$ are linearly independent on C^n for all ω_1, ..., $\omega_n \in \Re^n$. Then ϕ is PE.

Regressor vector $\phi_r(t)$ is given by

$$\phi_r(t) = \begin{bmatrix} \phi_{r,1} \\ \phi_{r,2} \end{bmatrix} = \begin{bmatrix} \omega(t) \\ -k_p e(t) - k_i \int_0^t e(\tau)d\tau \end{bmatrix} \qquad (25)$$

To prove that $\phi_r(t)$ is PE, consider the following transfer function related to the signals belonging to $\phi_r(t)$ defined in (25)

$$L(s) = \begin{bmatrix} L_1(s) \\ L_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{\Phi_{r,1}(s)}{Q_d(s)} \\ \dfrac{\Phi_{r,2}(s)}{Q_d(s)} \end{bmatrix} \qquad (26)$$
$$= \begin{bmatrix} \dfrac{bk_p s + bk_i}{s^2 + (a + bk_p)s + bk_i} \\ -\dfrac{(k_p s + k_i)(s + a)}{s^2 + (a + bk_p)s + bk_i} \end{bmatrix}$$

where $\Phi_{r,1}(s)$, $\Phi_{r,2}(s)$ and $Q_d(s)$ are the Laplace transforms of $\phi_{r,1}$, $\phi_{r,2}$ and q_d, respectively. We have that $(a+bk_p)>0$ and $bk_i>0$, then (26) is stable. To apply Proposition 1, assume that $q_d(t)$ is SR and consider the following complex matrix

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$$M = \left[L(j\omega_1) \quad L(j\omega_2) \right]$$
$$= \begin{bmatrix} \dfrac{bk_p(j\omega_1)+bk_i}{(j\omega_1)^2+(a+bk_p)(j\omega_1)+bk_i} & \dfrac{bk_p(j\omega_2)+bk_i}{(j\omega_2)^2+(a+bk_p)(j\omega_2)+bk_i} \\ -\dfrac{[k_p(j\omega_1)+k_i][(j\omega_1)+a]}{(j\omega_1)^2+(a+bk_p)(j\omega_1)+bk_i} & -\dfrac{[k_p(j\omega_2)+k_i][(j\omega_2)+a]}{(j\omega_2)^2+(a+bk_p)(j\omega_2)+bk_i} \end{bmatrix}$$

whose determinant is

$$\det(M) = \frac{(\omega_2-\omega_1)[bk_ik_p(\omega_2+\omega_1)]+j(\omega_2-\omega_1)(bk_p{}^2\omega_1\omega_2-bk_i{}^2)}{[(j\omega_1{}^2)+(a+bk_p)(j\omega_1)+bk_i]\,[(j\omega_2{}^2)+(a+bk_p)(j\omega_2)+bk_i]}$$

Then, $det(M)\neq 0$ if $\omega_1\neq\omega_2$, which implies that $L(j\omega_1)$, $L(j\omega_2)$ are linearly independent. Consequently, $\phi_r(t)$ is PE. Finally, since vector $\phi(t)$ converges to $\phi_r(t)$, thus $\phi(t)$ converges to a vector satisfying a PE condition and then $\tilde{\theta}$ converges exponentially to zero.

III. EXPERIMENTAL RESULTS

The servomechanism employed for the experiments consists of a DC brushed motor controlled through a Copley Controls power amplifier, model 413, configured in current mode. A Servotek tachometer model SA-7388-1 gives velocity measurements. A MultiQ-3 card from Quanser Consulting performs data adquisition. The card has 12 *bits* analog-to-digital and digital-to-analog converters with a voltage range of *±5 volts*. The Matlab-Simulink software operating with the WINCON software from Quanser Consulting serves as programming platform. Sampling period was set to *1ms*. For the proposed identification method, parameters were set to $k_p=5$, $k_i=2$, $\Gamma=diag[5000,100000]$, $\mu=10$. The servo was excited using a signal corresponding to two turns per second plus the block Band-Limited White Noise from Matlab-Simulink. The output of this block was subsequently filtered by a low pass filter with a cutoff frequency of *5Hz*. Fig. 02 shows the excitation signal. Fig. 03 depicts the signals ω and ω_e and parameter estimates $\hat{a}$ and $\hat{b}$ are shown in Fig. 04 and Fig. 05, respectively. Finally, Fig. 06 shows the error ε.

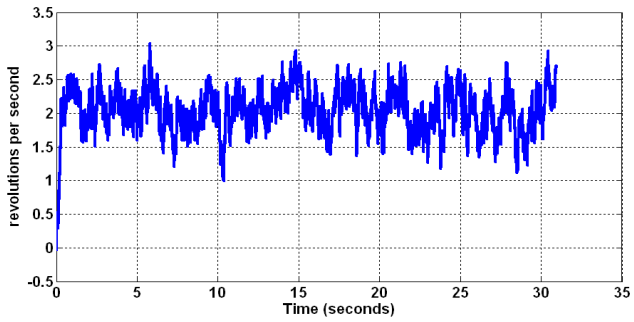

Fig. 2. Excitation signal ω_d.

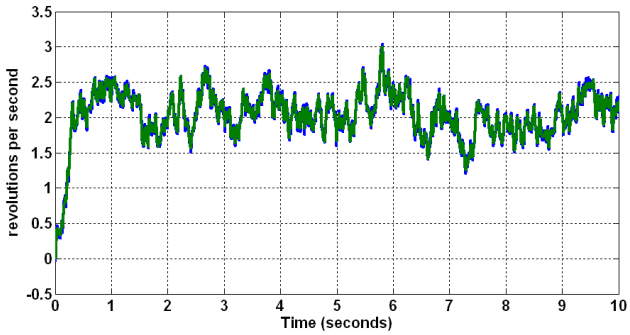

Fig. 3. Signals ω and ω_d.

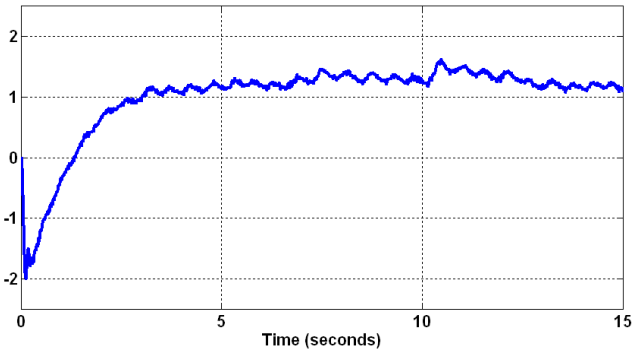

Fig. 4. Parameter estimated $\hat{a}$.

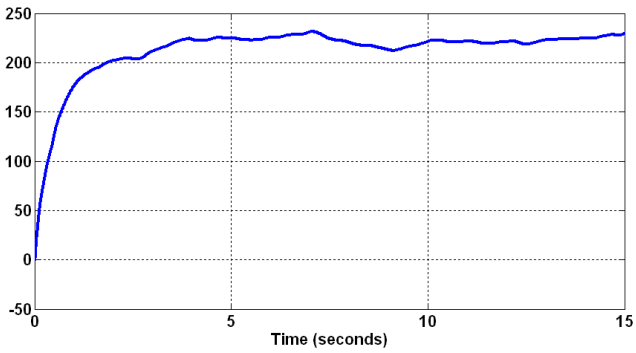

Fig. 5. Parameter estimated $\hat{b}$.

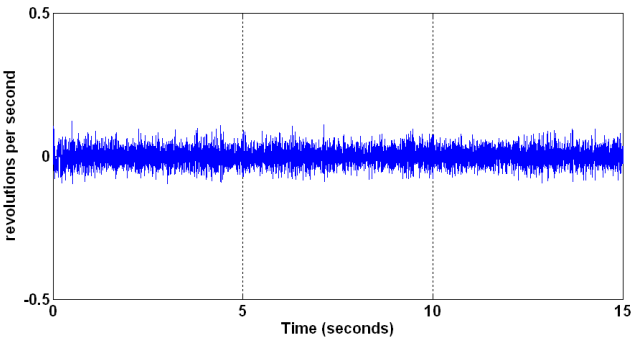

Fig. 6. Error ε.

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It can be seen that whitin a few seconds parameter estimates settles around constant values. It is also worth mentioning that the proposed parameter identification method worked out in spite of measurement noise and unmodelled dynamics. In the case of measurement noise, the tachometer employed during the experiments has a voltage ripple of 300mV. The unmodelled dynamics correspond to the electrical dynamics associated to the servomotor and the current amplifier.

IV. CONCLUSION

This paper presents preliminary results concerning a new methodology for closed-loop identification of servomotors. Compared with previous developments, the proposed approach allows estimating the servomotor parameters when it works in closed loop under velocity PI control. Experiments using a laboratory prototype shows the performance of the method. Future work includes comparing this methodology against methods published previously.

REFERENCES

[1] G. Ellis, *Control System Design Guide*, San Diego, USA: Academic Press, 2000.

[2] W. Lord, J. H. Hwang, "DC Servomotors: Modeling and parameter determination", *IEEE Trans. Industrial Applications*, Vol. 1A-13, no. 3, pp. 234-243, 1977.

[3] W. Lord, J. H. Hwang, "DC Servomotors: Modeling and parameter determination", *IEEE Trans. Industrial Applications*, Vol. 1A-13, no. 3, pp. 234-243, 1977.

[4] S. Daniel-Berhe, H. Unbehauen, "Experimental physical parameter estimation of a thyristor driven DC motor using the HMF method", *Control Engineering Practice*, vol. 6, pp. 615-626, 1998.

[5] S. Daniel-Berhe, H. Unbehauen, "Physical parameters estimation of the nonlinear continuous-time dynamics of a DC motor using Hartley modulating functions", *Journal of the Franklin Institute*, vol. 336, pp. 481-501, 1999.

[6] S. A. Soliman, A. M. Al-Kandari, M. E. El-Hawari, "Parameter identification method of a separately excited DC motor for speed control", *Electric Machines and Power Systems*, vol. 26, no. 8, pp. 831-838, 1998.

[7] Y. Y. Chen, P. Y. Huang, J. Y. Yen, "Frequency domain identification algorithms for servo systems with friction", *IEEE Trans. on Control Systems Technology*, vol. 10, no. 5, pp. 654-665, 2002.

[8] T. Kara, I. Eker, "Nonlinear closed loop identification of DC motor with load for low speed two directional operation", *Electrical Engineering*, vol. 86, no. 2, pp. 87-96, 2004..

[9] A. Krneta, S. Antić, Danilo Stojanović, "Recursive Least Squares Method in Parameter Identification of DC Motor Models", *Facta Universitatis, Series: Electronics and Energetics*, Vol. 18, N° 3, 2005.

[10] I. Eker, "Open loop and closed loop experimental on-line identification of a three mass electromechanical system", *Mecatronics*, vol. 14, pp. 549-565, 2004.

[11] M. Fliess, H. Sira-Ramirez, "Closed-loop parametric identification for continuous-time linear systems via new algebraic techniques. In *Continuous-Time Model Identification from Sampled Data*, H. Garnier & L. Wang Eds. Springer Verlag, 2008.

[12] *S. Sastry, M. Bodson, Adaptive Control, Stability, Convergence and Robustness,* Prentice-Hall*, 1989*.

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