

# Closed-loop identification analysis of a DC servomechanism: A Passive Approach

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Abstract — This paper deals with the analysis of a closedloop identification technique applied to a DC servomechanism from a passivity point of view. It is shown that the closed-loop system together with the identification algorithm can be divided into simpler subsystems easier to analyze and then, many properties related to passivity and stability can be deduced. Furthermore, it is shown that with this separation approach we have the freedom to select many controller structures, which could let to improve the performance of the identification algorithm, even in presence of perturbation signals.

*Keywords* — Closed-loop identification, passivity, persistent excitation

#### I. INTRODUCTION

System identification is an important tool which let us obtain important information that can be used to improve the performance of a system. There are several techniques and classifications in the literature to perform parameter identification. One possible classification for parameter identification is open-loop and closed-loop based techniques. In the first case, simplicity is an important feature which makes it attractive, unfortunately, such a techniques require the system to be open-loop stable and cannot be applied to unstable systems. Works in this vein are those based on the step response, the Least Squares (LS) method and the gradient method, as shown in references [6], [7]. However, there exist many situations where performing open-loop identification is dangerous or impossible. For example, when security is compromised and when it is impossible to remove the system controller, open-loop identification of the system parameters is not an alternative. examples are those which involve Some DC servomechanisms controlled in position mode, where there exists a pole at the origin, making the system marginally stable. Other examples are robotics systems, which possesses a controller and if it is removed, guaranty will cease. In such cases, closed-loop identification leads to a reasonable alternative because closing the loop allows perform system identification in safety conditions. Other classification is proposed in [6], considering direct and indirect methods for closed-loop system identification, where indirect methods consider the controller structure for identification purposes and will be considered in this work.

Another important feature to be considered in system identification is the Spectral Richness (SR) of the excitation signal used for the identification experiments or the so called Persistent Excitation (PE) condition [4]. It is well known that, for linear systems, if the regressor vector fulfills the PE condition, this ensures that the parameter error will converge to zero, then, in the identification framework it is very important to analyze under which conditions such a condition will be fulfilled.

On the other hand, the passivity theory gives a framework for the design and analysis of control systems using an input-output description based on energy-related considerations and can be used in many areas of science which yields to a modular approach to control systems design and analysis. When modeling passive systems, it may be useful to develop the state-space or input-output models so that they reflect the passivity properties of the system, and thereby ensure that the passivity of the model is invariant with respect to model parameters, and to the mathematical representation used in the model.

In this work the analysis of a closed-loop identification algorithm applied to a DC servomechanism is performed. The passivity approach is used to divide the whole system into three subsystems which are simpler to analyze and deduce its properties related to passivity, stability and parameter convergence. The objective is to show that with this separation, it is possible to have the freedom to select a class of linear or nonlinear controller, with some desired properties, and then obtain an identification algorithm with a good performance, i.e., a robust identification algorithm even in the presence of disturbances. Section II gives some preliminary results related with passivity theory. In Section III the system description, related with a DC servomechanism is presented. Section IV presents the passivity and stability analysis, considering as special cases the PD and PID controller. Finally, some concluding remarks are given in Section V.

## II. PRELIMINARY RESULTS

The main objective of this paper is to analyze a methodology for closed-loop identification of a DC servomechanism from a passivity viewpoint. In order to clarify the results and analysis that will be developed, the next definitions and results are important. These results were



mainly taken from references [1], [2] and [3]. For some results the Fig. 01 will be considered, the system  $\Sigma_a$  and the general nonlinear system  $\Sigma$  will also be considered and are defined as follows:

$$\Sigma_a \begin{cases} \dot{x} = f(x) + g(x)u, x(0) = x_0 \in \mathbb{R} \\ y = h(x) \end{cases}$$
(1)

$$\Sigma \begin{cases} \dot{x} = f(x, u), x(0) = x_0 \in \mathbb{R} \\ y = h(x, u) \end{cases}$$
(2)

Definition 1. ([3]) The system (2) is dissipative with respect to the supply rate  $\omega(u,y): \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$  if and only if there exist a storage function  $H: \mathbb{R}^n \to \mathbb{R}_{\geq 0}$  such that  $H(x(T)) \leq$  $H(x(0)) + \int_0^T \omega(u(t), y(t)) dt$  for all u, all  $T \geq 0$  and all  $x_0 \in \mathbb{R}^n$ .

Definition 2. ([3]) The system (2) is passive if it is dissipative with supply rate  $\omega(u,y)=u^T y$ . It is Input Strictly Passive (ISP) if it is dissipative with supply rate  $\omega(u,y)=u^T y$ .  $\delta_i ||u_i||^2$ , where  $\delta_i > 0$ . Finally, it is Output Strictly Passive (OSP) if it is dissipative with supply rate  $\omega(u,y)=u^T y$ .  $\delta_o ||y||^2$ , with  $\delta_o > 0$ .

Definition 3. ([3])  $\Sigma$  is said  $L_2$ -stable if there exists a positive constant  $\gamma$  such that for every initial condition  $x_0$ , there exists a finite constant  $\beta(x_0)$  such that  $||y||_{2T} \le ||u||_{2T} + \beta(x_0)$ .

Definition 4. A state space system  $\dot{x} = f(x)$ ,  $x \in \mathbb{R}^n$  is zero state observable (ZSO) from the output y=h(x), if for all initial conditions  $x(0) \in \mathbb{R}^n$  we have:  $y(t) \equiv 0 \Rightarrow x(t) \equiv 0$ . It is zero state detectable (ZSD) if:  $y(t) \equiv 0 \Rightarrow lim_{t \to \infty} x(t) = 0$ .

Definition 5. ([4]) A vector  $\phi: \mathbb{R}_+ \to \mathbb{R}^{2n}$  is persistently exciting (PE) if there exists positive constants  $\{\alpha_1, \alpha_2, \delta\}$  such that  $\alpha_1 I \leq \int_{t_0}^{t_0+\delta} \phi(\tau) \phi^T(\tau) d\tau \leq \alpha_2 I, \forall t_0 \geq 0.$ 

Definition 6. ([4]) A stationary signal  $r: \mathbb{R}^* \to \mathbb{R}$  is Sufficiently Rich (SR) or order k if the spectral density support of r has at least k points.

*Proposition 1*. ([3]) If  $\Sigma: u \rightarrow y$  is OSP, then it is L<sub>2</sub>-stable.

**Proposition 2.** ([3]) Consider the input-output system depicted in Fig. 01. If  $H_1$  and  $H_2$  are both passive, then the feedback interconnected system is also passive. If furthermore they are OSP, then the closed-loop system is also OSP.

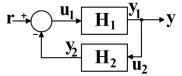


Figure 01. Closed-loop system with one external input

Theorem 1. ([1]) Assume that both  $H_1$  and  $H_2$  (see Fig. 01) fulfills the conditions:  $\int_0^t y_1^T u_1 dt + \beta_1 \ge \delta_1 \int_0^t y_1^T y_1 dt + \epsilon_1 \int_0^t u_1^T u_1 dt$  and  $\int_0^t y_2^T u_2 dt + \beta_2 \ge \delta_2 \int_0^t y_2^T y_2 dt + \epsilon_2 \int_0^t u_2^T u_2 dt$ , with  $(\delta_1 + \epsilon_1) > 0, (\delta_2 + \epsilon_2) > 0$ . The feedback closed-loop system is finite gain stable if  $\delta_2 \ge 0$ ,  $\epsilon_1 \ge 0, \epsilon_2 + \delta_1 > 0$ , where  $\epsilon_2$  or  $\delta_1$  may be negative.

Theorem 2. ([4]) Let  $\phi(t) \in \mathbb{R}^{2n}$  be the output of a Linear Time Invariant system with transfer function  $H_{\varphi r}(s)$  and stationary input r(t). Assume that  $H_{\phi r}(j\omega_1), \dots, H_{\phi r}(j\omega_{2n})$ are linearly independent in  $\mathcal{C}^{2n}$  for all  $\omega_1, \dots, \omega_{2n} \in \mathbb{R}$ . Then,  $\phi$ is PE if and only if *r* is SR or order 2*n*.

Theorem 3. ([3]) Suppose the system  $\Sigma_a$  is OSP with positive semidefinite storage function  $H \ge 0$ . (a) If  $\Sigma_a$  is ZSO, then  $H(x) \ge 0$  for all  $x \ne 0$ . (b) If  $H(x) \ge 0$  for all  $x \ne 0$ , H(0) = 0 and  $\Sigma_a$  is ZSD, then x=0 is a locally asymptotically stable equilibrium of  $\dot{x} = f(x)$ . Furthermore, if H is radially unbounded, the stability is global.

Corollary 1. ([1]) The feedback system in Fig. 01 is  $L_2$ finite gain stable if: (1)  $H_1$  is passive and  $H_2$  is ISP, i.e.,  $\epsilon_1 \ge 0, \epsilon_2 \ge 0, \delta_1 \ge 0, \delta_2 \ge 0$ ; (2)  $H_1$  is OSP and  $H_2$  is passive, i.e.,  $\epsilon_1 \ge 0, \epsilon_2 \ge 0, \delta_1 \ge 0, \delta_2 \ge 0$ .

Lemma 1. ([3]) Let y=G(p)u, where G(p) is an  $n \times m$  strictly proper, exponentially stable transfer function and p=d/dt. Then,  $u \in L_2^n$  implies that  $y \in L_2^n \cap L_{\infty}^n$ ,  $\dot{y} \in L_2^n$ , y(t) is continuous and  $y \rightarrow 0$  as  $t \rightarrow \infty$ . If, in addition,  $u \rightarrow 0$  as  $t \rightarrow \infty$ , then  $\dot{y} \rightarrow 0$ .

# **III. SYSTEM DESCRIPTION**

This paper considers the closed-loop parameter identification analysis of a DC servomechanism from a passivity point of view, where the servomechanism position will be considered as the output of the system. The methodology for closed-loop identification is the same than that presented in references [8], [9] and [10], where the idea of the identification algorithm is as follows: the real servomechanism is controlled using a Proportional Derivative (PD) or a Proportional Integral Derivative (PID) controller and it is considered a model of the real servomechanism whose feedback loop is closed using again a PD or a PID controller, where the same gains are employed in both controllers. Then, a gradient parameter identification algorithm is employed for estimating the system parameters and the estimated parameters update the model of the real servomechanism. We will consider the model of the DC servomechanism without any perturbation signal, so that its model can be given as:

$$\ddot{q}(t) = -a\dot{q}(t) + bu(t) \tag{3}$$

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with q(t) being the position of the servomechanism,  $\{a,b\}$  are the system parameters (which are assumed to be constant) and u(t) is the control input. The analysis that will be presented will consist on separating the whole system into subsystems, analyze the passivity properties of each subsystem and then put them together, so that the first analysis can be used to deduce stability properties for the whole system. Besides, for closing the loop, a PD and a PID controller will be considered.

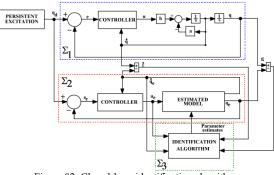


Figure 02. Closed-loop identification algorithm

An important feature of this methodology is that stability of the closed-loop with the real servomechanism can be claimed without requiring the knowledge of the system parameters; for instance, it is easy to show that the system (3) with a PD controller leads to a polynomial which can be analyzed by using the Routh-Hurwitz criterion and then, conclude stability without regarding the value of the system parameters. The idea of the closed-loop identification algorithm is depicted in Fig. 02, where a model of the servomechanism with output  $q_e(t)$  is considered and its dynamic equation is given by:

$$\ddot{q}_e(t) = -\hat{a}\dot{q}_e(t) + \hat{b}u_e(t) \tag{4}$$

where the parameters  $(\hat{a}, \hat{b})$  denote the estimates of (a, b). Let to define the output error  $\epsilon(t) = q - q_e$ . Then, by taking the second time derivative of  $\epsilon(t)$  and using (3) and (4) we get:

$$\ddot{\epsilon} + a\dot{\epsilon} = b\epsilon_c + \tilde{\theta}^T \phi \tag{5}$$

where  $\phi(t) = (\dot{q}_e, -u_e)^T$  is the so called regressor vector and  $\tilde{\theta}(t) = (\hat{a} - a, \hat{b} - b)^T = \hat{\theta}(t) - \theta$  is the parameter error vector,  $\hat{\theta}(t)$  denote the estimated parameters vector,  $\theta$ the real parameter vector and  $\epsilon_c = u(t) - u_e(t)$ .

# IV. PASSIVITY ANALYSIS

In this section the system (5) will be split into three subsystems and some passivity properties will be inferred from each subsystem. To this end, let to consider the input  $u_1 = b\epsilon_c + \tilde{\theta}^T \phi$ . Then, (5) can be seen as a system  $\Sigma_1$  with input  $u_1$  and output  $y_1$  as in the next equation:

$$\Sigma_1: \ddot{\epsilon} + a\dot{\epsilon} = u_1 \tag{6}$$

Now, the controller used for closing the loop of the system can be considered as another system with a specific dynamic equation. This viewpoint is important because we can analyze the controller as a second system which posses some passivity properties and then apply some useful results, mainly that stated in Proposition 2, which says that the negative feedback interconnection of two passive systems remains passive. Besides, another important result that can be drawn is that if we consider the real system and the controller as Euler Lagrange (EL) systems, then it is possible to interconnect them so that the resulting feedback system is an EL system, whose EL parameters are simply the sum of the EL parameters of each subsystem [3]. Therefore, let us consider a second subsystem  $\Sigma_2$  with state  $x_2(t)$  and a third subsystem  $\Sigma_3$  with state  $x_3 = \tilde{\theta}(t)$ , where the general state-space description for the system  $\Sigma_i$  can be given as follows:

$$\Sigma_{i} \begin{cases} \dot{x}_{i} = f_{i}(x_{i}) + g_{i}(x_{i})u_{i} \\ y_{i} = h_{i}(x_{i}) + j_{i}(x_{i})u_{i} \end{cases}, i = 2,3$$
(7)

Then,  $\Sigma_2$  can be seen as a second subsystem with input  $u_2 = y_1$  and output  $y_2 = -b\epsilon_c$  and, in order to perform the parameter identification, we can consider the system  $\Sigma_3$  with Input  $u_3$ , to be defined later, and output  $y_3 = -\tilde{\theta}^T \phi$ . Therefore, by using the three subsystems presented above, it is possible to consider the whole system (5) as the negative feedback interconnection of these subsystems as depicted in Fig. 03. Now we have a specific form for each subsystem, it is possible to analyze them separately. To this end, let us consider first the subsystem (6) and the storage function  $H_1$ as follows:  $H_1(\epsilon, \dot{\epsilon}) = \frac{\mu a}{2} \epsilon^2 + \frac{1}{2} \dot{\epsilon}^2 + \mu \epsilon \dot{\epsilon}$ , which is positive definite if  $\mu < a$ , where  $\mu \in \mathbb{R}^+$ . Then, by taking the time derivative of  $H_1$  and considering (6) yields  $\dot{H}_1 =$  $\mu a \epsilon \epsilon + \dot{\epsilon} \ddot{\epsilon} + \dot{\mu} \epsilon \ddot{\epsilon} + \mu \dot{\epsilon}^2 \le u_1(\mu \epsilon + \dot{\epsilon}), \text{ if } \mu < a \text{ holds.}$ Thus, the system  $\Sigma_1$  defines the passive mapping:  $u_1 \rightarrow y_1$ , with output  $y_1 = (\mu \epsilon + \dot{\epsilon})$ . At this point, it is useful to consider Corollary 1 and Lemma 1 because, from the structure of the output  $y_1(t)$  and using the Laplace transform with  $\epsilon(s) = \mathcal{L}\{\epsilon(t)\}, y(s) = \mathcal{L}\{y(t)\}$  and  $\mathcal{L}\{\cdot\}$ the Laplace operator, it is easy to see that:  $\epsilon(s) =$  $(s + \mu)^{-1} y(s)$ , which means that  $\epsilon(t)$  corresponds to the output of an exponentially stable transfer function, then we can search for a feedback interconnection such that the resulting system will be  $L_2$ -finite gain stable (i.e., an ISP system according to Corollary 1), and then  $\epsilon(t) \xrightarrow{t \to \infty} 0$ follows.

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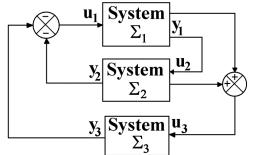


Figure 03. Negative feedback interconnection of three subsystems

Therefore, one possibility is to consider a controller  $\Sigma_2$  which posses the ISP property. Note that by setting  $y_1 \equiv 0$ , from (6) we get that  $\epsilon = 0$ ,  $\dot{\epsilon} = 0$ , i.e., the system (6) is ZSO, and this property, together with the positive definiteness of  $H_1$ , can be used to conclude stability of this system, as mentioned in Theorem 3. Finally, in order to show explicitly the properties of passivity and convergence we will consider the special case of the PD and PID controllers as follows.

# A. PD Controller Analysis

Let us consider the case of the PD controller u(t) for the real servomechanism and the controller  $u_e(t)$  for the estimated model, described by:  $u(t) = k_p e(t) - k_d \dot{q}(t), u_e(t) = k_p e_e(t) - k_d \dot{q}_e(t)$ , where  $e(t) = q_d - q, e_e = q_d - q_e$ , with  $\{q, q_e\}$  being the outputs from the real servomechanism and its model, respectively. Note that the same gains  $\{k_p, k_d\}$  are used for both controllers. Then, it can be verified that  $\epsilon_c = -k_p \epsilon - k_d \dot{\epsilon}$ . From the interconnection shown in Fig. 03 and the definition of  $y_1(t)$ , we have  $u_2 = (\mu \epsilon + \dot{\epsilon})$ . Then, we have the state  $x_2 = \epsilon$ , thus, the dynamics of the controller is given by:

$$\Sigma_{2} \begin{cases} \dot{x}_{2} = -\mu x_{2} + u_{2} \\ y_{2} = (bk_{p} - \mu bk_{d})x_{2} + bk_{d}u_{2} \end{cases}$$
(8)

Now, let us assume that  $(bk_p - \mu bk_d) = \gamma > 0$ . Consider the positive definite storage function  $H_{2-PD} = \frac{1}{2}\gamma x_2^2$ . Then, using (8) it is possible to obtain:

$$\dot{H}_{2-PD} \le u_2 y_2 - b k_d u_2^2$$

therefore, we conclude that the controller dynamics (8) describes the ISP operator  $u_2 \rightarrow y_2$ . Thus, from Corollary 1 we conclude that the feedback interconnection of  $\Sigma_1$  and  $\Sigma_2$  is  $L_2$  finite gain stable, therefore,  $y_1 \in L_2$ . Now, from definition of  $y_1$ , it is possible to note that  $\epsilon$  corresponds to the output of a strictly proper, exponentially stable transfer function, then, from Lemma 1 we conclude that  $\epsilon(t) \xrightarrow{t \to \infty} 0$ .

# B. PID Controller Analysis

As in the previous case, let us consider the PID controllers:  $u(t) = k_p e(t) - k_d \dot{q}(t) + k_i \int_0^t e d\tau$  and  $u_e(t) = k_p e_e(t) - k_d \dot{q}_e(t) + k_i \int_0^t e_e d\tau$ , with the same definitions for all the signals involved. In this case we have  $\epsilon_c = -k_p \epsilon - k_d \dot{\epsilon} - k_i \int_0^t \epsilon d\tau$ . As in the PD case, we have the controller input  $u_2 = (\mu \epsilon + \dot{\epsilon})$ , but now the state is  $x_2 = (x_{21}, x_{22})^T = (\int_0^t \epsilon d\tau, \epsilon)^T$ . Therefore, with the same definition for  $\gamma$  in the PD case, the dynamics of the PID controller is given by:

$$\Sigma_{2} \begin{cases} \dot{x}_{21} = x_{22} \\ \dot{x}_{22} = -\mu x_{22} + u_{2} \\ y_{2} = bk_{i} x_{21} + \gamma x_{22} + bk_{d} u_{2} \end{cases}$$
(9)

In this case we consider the storage function:

$$H_{2-PID} = \frac{\mu b k_i}{2} x_{21}^2 + b k_i x_{21} x_{22} + \frac{\gamma}{2} x_{22}^2 \qquad (10)$$

which will be positive definite if the condition  $k_p \ge (\mu k_d + 2k_i/\mu)$  holds. Now, the time derivative of (10) along the trajectories of (9) is:

$$\dot{H}_{2-PID} \le u_2 y_2 - b k_d u_2^2$$

thus we conclude that the system (9) defines the ISP mapping:  $u_2 \rightarrow y_2$  and, following the same lines as in the PD case, it is possible to conclude that  $\epsilon(t) \xrightarrow[t \rightarrow \infty]{} 0$ .

### C. Identification algorithm

By the structure presented above, there is some freedom for selecting the identification algorithm. One possibility is the gradient algorithm presented in [8]. To represent such algorithm in the form (7), let to define  $x_3 = \tilde{\theta}$  and the input  $u_3 = y_1$ . Then, the identification algorithm dynamics is given by:

$$\Sigma_3 \begin{cases} \dot{x}_3 = -\Gamma \phi u_3 \\ y_3 = -\phi x_3 \end{cases}$$
(11)

In this case, we can consider the following storage function:  $H_3 = \frac{1}{2} x_3^T \Gamma^{-1} x_3$ , whose time derivative along the trajectories of (11) is:  $\dot{H}_3 = x_3^T \Gamma^{-1} \dot{x}_3 = u_3^T y_3$ , which shows that the system (11) describes a passive operator:  $u_3 \rightarrow y_3$ . This property will let us interconnect this system with the system  $\Sigma_1$  and, then, use the passivity properties of the feedback system using again the passivity invariance property stated in Proposition 2, as will be shown later.

### D. Stability Analysis

The interconnection presented in Fig. 03 is important because of the invariance of passivity property described in Proposition 2 and the feedback interconnection properties described in Theorem 1 and Corollary 1. Note that the system (5) can be obtained if we first make the feedback interconnection of  $\Sigma_1$  and  $\Sigma_3$ . Then, this resulting system can be interconnected again in the same way with  $\Sigma_2$ . The first interconnection is passive according with Proposition 2, while the second interconnection is  $L_2$ -finite gain stable according with Corollary 1. Therefore, for the whole system we have that  $y_1 \in L_2$  and it can be concluded that  $\epsilon(t)$  tends to zero as t tends to infinity, as mentioned before, using Lemma 1. However, there are other important features that we can analyze from the structure presented. For instance, the sum of all the storage functions can be considered as a Lyapunov function, although in this case we split the system into simpler subsystems, then, it was straightforward to analyze them independently and get conclusions for the whole system in a simpler way. For example, for the PD case it is possible to consider the following positive definite Lyapunov function:  $V_1 = H_1 + H_{2-PD} + H_3$ , whose time derivative along the trajectories of (6) can be upper bounded as:  $\dot{V}_1 = \leq -(a - \mu)\dot{\epsilon}^2 - \mu\gamma\epsilon^2$ , which shows that  $V_1 < 0$ , then, it is easy to see that all the signals remain bounded and that  $\epsilon(t) \in L_2$ . Besides, if  $y_1 \equiv 0$ , then, we can conclude that (6) is ZSO. Then, the system equilibrium is asymptotically stable. Thus q(t) and  $q_e(t)$  are bounded. Also,  $u_1 =$  $-(y_2 + y_3) \in L_{\infty}$ , then, from (6) we have that  $\ddot{\epsilon} \in L_{\infty}$ , therefore, from Lemma 1 we conclude that  $\dot{\epsilon} \xrightarrow[t \to \infty]{} 0$ . A second important property is that the Euler-Lagrange structure is preserved. From this point of view, it is possible to consider the potential and kinetic energy of each subsystem for control design [3]. In this way, we can apply the Passivity Based Control (PBC) approach [1], [2], [3] in order to design a controller that modifies the open loop energy of the system in a desired way. For example, the potential energy can be modified for having a unique minimum at a desired point  $q_d$  for regulation purposes or both, the potential and kinetic energy can be modified in order to accomplish trajectory tracking. This approach is a key feature presented in [3] that can be useful if we employ the passivity based approach. The last property to be considered is that related with the parameter convergence. From the stability analysis presented above, let us recall that  $u_{e}(t)$  is an element of the regressor vector  $\phi(t)$ . This is important because as mentioned in [5] and [10], the structure of the controller influences the parameter convergence. For example, we have that nonlinearities enhance parameter convergence and reduce the variance of parameter estimates in presence of disturbance signals. Therefore, one aspect to consider when designing the controller is which structure would improve the performance of the parameter identification algorithm. For example, in

[10] the authors showed that the performance of the identification algorithm can be increased through the effect of the gains of the controllers and that a PID controller can

identification algorithm can be increased through the effect of the gains of the controllers and that a PID controller can lead to best estimates than those obtained using only a PD controller. On the other hand, if nonlinearities enhance parameter convergence, then it is possible to design a controller which fulfills the ISP condition presented above, but with some nonlinearity that benefit the parameter convergence; then, the next step that could be further investigated is which class of controllers can be used to improve the performance of the identification algorithm. As an example of the simplicity for the structure of subsystems presented above, consider the gradient algorithm used in [10] for parameter estimation. In this case the regressor vector is given by the following structure  $\phi(t) = (\dot{q}_e, u_e)^T$ and a PD controller is considered. Then, by using the regressor vector  $\phi_r(t) \triangleq (\phi_{r1}, \phi_{r2})^T = (\dot{q}, -u)^T$ , then it can be shown that:

$$\frac{\Phi_{r1}}{Q_d} = \frac{bk_ps}{s^2 + cs + bk_p}, \frac{\Phi_{r2}}{Q_d} = \frac{-k_ps(s+a)}{s^2 + cs + bk_p}$$

where  $\Phi_{r1} = \mathcal{L}\{\phi_{r1}(t)\}, \quad \Phi_{r2} = \mathcal{L}\{\phi_{r2}(t)\}$  and  $Q_d = \mathcal{L}\{q_d(t)\}$ . Thus, from Theorem 2 it is possible to conclude that if the reference  $q_d$  is PE, then  $\phi_r(t)$  will be PE and, from the stability analysis presented above, it can be concluded that  $\phi(t)$  converges to a vector that fulfills the PE conditions, then, ensuring that the parameter error vector  $\tilde{\theta}(t)$  converges to zero. Besides, another important feature that we can remark here is the freedom for selecting any input signal that accomplish the PE condition (or the SR condition of Definition 6). Therefore, we can select both the controller and the excitation signal in order to find out a better class of identification algorithms whose performance rely not only on the excitation signal and the identification algorithm gains, but also in the controller structure and its gains. The same conclusions for the stability analysis and parameter convergence can be drawn for the PID case. Finally, it is clear that in practice not only nonlinearities can be a drawback for identification purposes, but also the perturbation signals. However, this could not be actually a significant problem if we consider the Theorem 10 presented in [10] and stated as folows.

Theorem 4. ([10]) Consider the perturbed system  $\ddot{e} = -c\dot{e} - bk_p\epsilon + \tilde{\theta}^T\phi + \nu$ , with  $\nu$  a perturbation signal. If the equilibrium  $w_0$  of the unperturbed system is exponentially stable, then: (i) The perturbed system is small signal  $L_{\infty}$  stable, that is, there exists  $\gamma_{\infty}$  such that  $||w(t)|| \leq \gamma_{\infty}\beta < h$ , where w(t) is the solution of the perturbed system starting at  $w_0$ . (ii) There exists  $m \geq 1$  such that  $||w_0|| < h/m$  implies that w(t) converges to a ball  $\Omega_{\delta}$  of radius  $\delta = \gamma_{\infty}\beta < h$ , that is: for all  $\varepsilon > 0$  there exists  $T \geq 0$  such that  $||w(t)|| \leq \delta(1 + \varepsilon)$  for all  $t \geq T$ , along the solutions of the perturbed system starting at  $w_0$ . Also, for all  $t \geq 0$ ,  $||w_0|| < h$ .

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The importance of this result is that it let us conclude that the controller gains actually reduce the size of the region  $\Omega_{\delta}$ , which gives robustness to the identification algorithm when there exist disturbances.

As a final step, the experimental result for closed-loop identification of a DC servomechanism is presented. The servomechanism employed for the experiments consists of a DC brushed Clifton Precision motor, model JDTH-2250-BQ-IC, driven by a Copley Controls analog power servoamplifier, model 413, configured in current mode. An optical encoder gives angular position measurements. A MultiQ-3 card from Quanser Consulting performs data acquisition. The Matlab/Simulink software operating with the WINCON software from Quanser Consulting serves as programming platform. The sampling period for all the experiments was set to 1 ms. The MatLab/Simulink block Band Limited White Noise provides the signal excitation and its parameters were set to Noise Power = 0.005, seed [1212121] and Sampling time = 0.1 s. The parameters of the real servomechanism (nominal parameters), were computed using the technical data of the servomotor and the power amplifier. The nominal values are a = 0.2174 and b = 257.7. A PID controller was employed with gains  $k_p = 9$ ,  $k_d =$ 0.19 and  $k_4 = 4$ . The update law gains were set to  $\Gamma =$  $diag\{0.2,110\}$  and  $\mu = 10$ . Performance of the proposed approach was compared against a recursive discrete-time Least Squares algorithm with forgetting factor [6]. In this case, a relay closes the loop and the regression model was obtained in the same way as in [9] by filtering the servomechanism input and ouput. Fig 04 (a) shows the estimated parameter  $\hat{a}$  for the proposed method and Fig. 04 (b) the same parameter obtained with the LS algorithm, while Fig. 04 (c) shows the estimated parameter  $\hat{b}$  for the proposed method and Fig. 04 (d) the same estimated parameter for the LS algorithm. From this figures it is possible to see that both identification algorithms give similar results, but the proposed method has a faster convergence time and gives estimates closer to the nominal parameters.

#### V. CONCLUSION

In this paper a passive analysis for closed-loop identification of a DC servomechanism was presented. It was shown that the closed-loop system with the identification algorithm can be split into three subsystems, which are simpler to analyze and then, passivity and stability properties can be obtained for the whole system employing the analysis performed on each subsystem. The advantage of using the passivity based approach is that it let us visualize a possible complex system as a set of simpler interconnections and then, decide which structure is appropriate for identification purposes. Besides, it was shown that the controller structure is an important feature that has not been underlined and that could be useful not

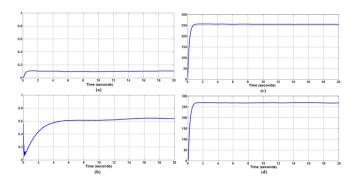


Fig. 04. Identified parameters: (a) Estimate  $\hat{a}$  for the proposed method; (b) estimate  $\hat{a}$  for the LS algorithm; (c) identified parameter  $\hat{b}$  for the proposed method; (d) identified parameter  $\hat{b}$  for the LS algorithm.

only for the performance of the controlled system, but also for ensuring the PE condition on the regressor vector and for enhancing parameter convergence.

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