Analysis of the Controller and Identification Algorithm Gains for Closed-Loop Identification Applied to a Perturbed DC Servomechanism working under PD control

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Abstract — **Usually, when an identification algorithm is applied to a given system, it is normal to pay attention only to the gains of the matrix related with the identification algorithm. However, it can be shown that, for some closed- loop identification structures, as the one studied in this paper, the gains of the controller employed for closing the loop of the system play an important role in the identification procedure and affect the region where the parameter estimates are supposed to converge to. In this situation it is important to consider the behavior that the parameter estimates have when varying not only the identification matrix gains, but also the controller gains. This paper analyses the behavior of the parameter estimates for a closed-loop identification methodology applied to a DC servomechanism with a bounded perturbation signal. Some properties of the region where the parameter estimates converge to are shown through the variation of both the PD controller gains and the identification matrix gains.**

Keywords — **closed loop identification, persistence of excitation, DC servomechanism.**

I. INTRODUCTION

Nowadays, Robotics is a prominent research field in all over around the world. The application of robotic systems is extending to most of the fields of our lives, for example, medicine, industry, research laboratories, etc. As expected, the variety of applications of such systems requires an improved performance which is obtained with the design of modern controllers that are able to overcome the problems related with unmodeled dynamics, perturbation signals, measurements noise and also with the system parameter uncertainties. Besides, it is well known that in many cases the DC servomotors are an important element for the actuation of such robotic systems. Therefore, the most we know about them the best performance the overall robotic system will have. However, there are many techniques for control which can give a very good performance when applied to a given system, but many of such techniques require the knowledge of the system parameters, which makes necessary to deal with parameter identification techniques.

In the case of parameter identification, there exist many techniques which perform it for a system working in open loop and closed loop, but in most industrial systems there is an inherent controller which cannot be removed for security reasons or because of the warranty. Besides, it is important to note that if the variable of interest is servo position, then, a linear model of a servomechanism contains a pole on the imaginary axis, thus making the system not BIBO stable, i.e., a bounded input applied to the servomechanism would not produce a bounded position. Therefore, for security reasons, parameter identification should be performed in closed loop. Thus, the closed loop identification is a necessary requirement if we want to design and apply a controller which gives a good performance on the system under study.

References [3], [4], [5], [6] propose methods for closed loop identification of position-controlled servos where the loop is closed using a linear controller. The approach proposed in [6] uses a PD controller to close the loop and an on-line gradient algorithm allows estimating a linear model of a servomechanism. Relay-based techniques are widespread for servo identification [7], [8]. The idea behind these methods is to close the loop through a relay in order to obtain a sustained oscillation. Then, its amplitude and frequency allows identifying linear model and non linear models of a servomechanism.

Nearly all of the identification procedures using a linear controller [3], [4], [5] fall into the category of direct methods [9], i.e., the parameter identification procedure is applied without regard about the controller being used to close the loop. Moreover, most of the parameter identification techniques employ the Least Squares technique, which would give incorrect estimates if the disturbances affecting the servo have not zero mean. On the other hand, the relay-based methods give consistent results but tuning of the relay controller can be cumbersome. Besides, no one of the reviewed methods takes explicitly into account disturbances affecting the servomechanism.

This work presents a closed loop identification methodology applied to a perturbed position-controlled DC servomechanism where a PD controller is used to close the loop, achieving stability of it without knowledge about the servomechanism parameters. Theoretical results show that when the perturbation signal is identically zero, exponential convergence can be claimed, and in the presence of a bounded perturbation signal a region Ω_δ can be found where

IEEE Catalog Number: CFP11827-ART
ISBN: 978-1-4577-1013-1

the estimated parameters belong to, which can be made arbitrarily small if a high gain controller is employed. Besides, a further study is made on some equations which result from the stability analysis, which give an insight about how the controller gains have also some influence to reduce the region where the parameter estimates converge to in the presence of a bounded perturbation signal. The paper is organized as follows. Section II presents some theoretical results of parameter estimation and passivity based control taken from [2] and [12]. Section III presents a general description of the disturbed model to be identified. Section IV studies the non perturbed case for the identification algorithm and proves the exponential convergence of all the signals involved in the identification procedure, especially those corresponding to the parameter convergence. The perturbed case is analyzed in Section V and finally Section VI presents the analysis of the results. Section VII gives some concluding remarks.

II. PRELIMINARY RESULTS

The analysis of the identification algorithm presented in the next Section is based mainly on passivity based arguments, then, it is important to first introduce some preliminary results taken mainly from [2] and [12]. Besides, some important results related with the convergence of the parameter estimates taken from [11] are also presented.

Let consider the linear time-varying system $[C(t), A(t)]$ defined by:

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t), y(t) = C(t)\mathbf{x}(t) \qquad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^n, y(t) \in \mathbb{R}^m$, while $A(t) \in \mathbb{R}^{nXn}, C(t) \in \mathbb{R}^{mXm}$, are piecewise continuous functions.

Definition 1: (Uniform Complete Observability, UCO [11]) The system $[C(t), A(t)]$ is called uniformly completely observable (UCO) if there exist strictly positive constants β_1, β_2, δ, such that, for all $t_0 \geq 0$: $\beta_1 I \leq N(t_0, t_0+\delta) \leq \beta_2 I$, where $N(t_0, t_0+\delta) \in \mathbb{R}^{nXn}$ is the so-called observability grammian: $N(t_0, t_0+\delta) = \int_{t_0}^{t_0+\delta} \Phi^{\mathrm{T}}(\tau, t_0)\, C^T(\tau) C(\tau) \Phi(\tau, t_0) d\tau$.

Lemma 2: (UCO under output injection [11]) Assume that, for all $\delta > 0$, there exists $k_d > 0$ such that, for all $t_0 \geq 0$: $\int_{t_0}^{t_0+\delta} \| K(\tau) \|^2 \, d\tau \leq k_d$. Then, the system $[C, A]$ is UCO if and only if $[C, A+KC]$ is UCO. Moreover, if the observability grammian of the system $[C, A]$ satisfies the inequality for N in the UCO condition, then the observability grammian of the system $[C, A+KC]$ satisfies these inequalities with identical δ and $\beta_1' = \frac{\beta_1}{(1+\sqrt{k_\delta})^2}, \beta_2' = \beta_2 \exp(k_\delta \beta_2)$.

Theorem 3: ([11]) Assume that $f(t, \mathbf{x})$: $\mathbb{R}^+ \mathrm{X}\, \mathbb{R}^n \to \mathbb{R}^n$ has continuous and bounded first partial derivates in $\mathbf{x}$ and is piecewise continuous in t for all $\mathbf{x} \in B_h, t \geq 0$. Then the following statements are equivalent: (1) $\mathbf{x} = 0$ is an exponentially stable equilibrium point of $\dot{\mathbf{x}} = f(t, \mathbf{x}), \mathbf{x}(t_0) = \mathbf{x}_0$. (2) There exists a function $V(t, \mathbf{x})$ and some strictly positive constants $h', \propto_1, \propto_2, \propto_3, \propto_4$, such that, for all $\mathbf{x} \in B_h, t \geq 0$: $\propto_1 \| \mathbf{x} \|^2 \leq V(t, \mathbf{x}) \leq \propto_2 \| \mathbf{x} \|^2$, $\left.\frac{dV(t,\mathrm{x})}{dt}\right|_{\dot{\mathbf{x}}=f(t,\mathbf{x})} \leq -\propto_3 \| \mathbf{x} \|^2$ and $\left|\frac{\delta V(t,x)}{\delta x}\right| \leq -\propto_3 \|\mathrm{x}\|$.

Theorem 4: ([11]) if there exists a function $V(t, \mathbf{x})$ and strictly positive constants $\propto_1, \propto_2, \propto_3$ and δ, such that for all $\mathbf{x} \in B_h, t \geq 0$: $\propto_1 \| \mathbf{x} \|^2 \leq V(t, \mathbf{x}) \leq \propto_2 \| \mathbf{x} \|^2$, $\left.\frac{dV(t,\mathrm{X})}{dt}\right|_{\dot{\mathbf{x}}=f(t,\mathbf{x})} \leq 0$, $\int_t^{t+\delta} \frac{dV}{d\tau} V(\tau, \mathrm{x}(\tau))\Big|_{\dot{\mathbf{x}}=f(t,\mathbf{x})} d\tau \leq -\propto_3 \| \mathrm{x}(t) \|^2$, then $\mathbf{x}(t)$ converges exponentially to zero.

Theorem 5: [1] The state equation $[A, b, C, d]$ is a minimal realization of a proper rational function $\hat{g}(s)$ if only if (A, b) is controllable, (A, C) is observable or if and only if $\dim(A) = \deg\, \hat{g}(s)$, where $\hat{g}(s) = N(s)/D(s)$ and $\deg(\hat{g}(s)) = \deg(D(s))$.

In the next definitions, let to consider the system Π defined as

$$\Pi\colon \dot{\mathrm{x}}(t) = f(\mathbf{x}, u), x(0) = \mathbf{x}_0 \in \mathbb{R}^n \qquad (2)$$
$$y(t) = H(\mathbf{x}, u)$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^n$ the input and $y \in \mathbb{R}^m$ the system output. Consider the set Ξ of $n-$dimentional real valued functions $f(t)$: $\mathbb{R}^+ \to \mathbb{R}^n$ and define the set $L_2 \triangleq \left\{\mathbf{x} \in \Xi\colon \| f \|_2^2 \triangleq \int_0^\infty \| f(t) \|^2 \, dt < \infty\right\}$, with $\|\cdot\|$ the Euclidean norm. Such a set constitutes a normed vector space with the field $\mathbb{R}$ and norm$\|\cdot\|_2$. Let introduce the extended space L_{2e} as: $L_{2e} \triangleq \left\{\mathbf{x} \in \Xi\colon \| f \|_{2T}^2 \triangleq \int_0^T \| f(t) \|^2 \, dt < \infty, \forall T\right\}$, where $L_2 \subset L_{2e}$. In the same way let us introduce the inner product and the truncated inner product of the functions u and y as $(u, y) \triangleq \int_0^\infty u(t)^T y(t) dt, (u, y)_T \triangleq \int_0^T u(t)^T y(t) dt$.

Definition 6: (Dissipativity, [2]) The system Π is dissipative with respect to the supply rate $\omega(u, y)$: $\mathbb{R}^m \mathrm{X}\, \mathbb{R}^m \to \mathbb{R}$ if and only if there exists a storage function $H\colon \mathbb{R}^n \to \geq 0$, such that $H(\mathbf{x}(T)) \leq H(\mathbf{x}(0)) + \int_0^T \omega(u(t), y(t)) dt$ for all $u(t)$, all $T \geq 0$ and all $\mathbf{x}_0 \in \mathbb{R}^n$.

Definition 7: (Passivity,[2],[12]) The system Π is passive if its supply rate given by $\omega(u, y) = u^T y$. It is input strictly passive (ISP) if there exists a positive constant δ_i such that the supply rate can be expressed as $\omega(u, y) =$

$u^T y - \delta_i \parallel u \parallel^2$, $\delta_i > 0$. Finally, it is output strictly passive (OSP) if there exists a positive constant δ_0 such that the supply rate can be expressed as $\omega(u, y) = u^T y - \delta_0 \parallel u \parallel^2, \delta_0 > 0$.

Definition 8: (L_2 Stability,[2]) The system Π is called L_2 finite gain stable if there exists a positive constant γ such that for any initial condition $\mathbf{x}_0$ there exists a finite constant $\beta(\mathbf{x}_0)$ such that $\|y\|_{2T} \leq \gamma \|u\|_{2T} + \beta(\mathbf{x}_0)$.

Proposition 9: (OSP implies L_2stability, [2]) If $\Sigma: u \rightarrow y$ is OSP, then it is L_2 stable.

Definition 10: (Zero state Observability, [2],[12]) A state space system $\dot{\mathbf{x}}(t) = f(\mathbf{x}), \mathbf{x} \in \mathbb{R}^n$, is zero state observable from the output $y(t) = h(\mathbf{x})$ if for all initial conditions $\mathbf{x}(0) \in \mathbb{R}^n$ we have $y(t) \equiv 0 \Longrightarrow \mathbf{x}(t) \equiv 0$. It is zero state detectable if $y(t) \equiv 0$ implies that $\lim \mathbf{x}(t) = 0$ as $t \rightarrow \infty$.

Definition 11: (Persistently excitation, PE [11]) A vector ϕ: $\mathbb{R}^+ \rightarrow \mathbb{R}^{2n}$ is persistently excitation, (PE) if there exist constants $\{\propto_1, \propto_2, \delta\} > 0$ such that $\propto_1 I \leq \int_{t_0}^{t_0+\delta} \phi(\tau)\phi^T(\tau) d\tau \leq \propto_2 I \ \forall t_0 \geq 0$.

Lemma 12: (PE trough a LTI filter, [11]) Let ω: $\mathbb{R}^+ \rightarrow \mathbb{R}^{2n}$. If ω is PE, the signals ω,$\dot{\omega}$ belongs to the space L_∞ and H is a rational stable strictly proper minimum phase transfer function, then $H(\omega)$ is PE.

Theorem 13: (Small Signal I/O Stability,[11]) Consider the perturbed system $\dot{\mathbf{x}} = f(t, \mathbf{x}, u), \mathbf{x}(0) = \mathbf{x}_0$ and the unperturbed system $\mathbf{x} = f(t, \mathbf{x}, 0), \mathbf{x}(0) = \mathbf{x}_0$, where $t \geq 0, \mathbf{x} \in \mathrm{B_h}, \mathrm{u} \in \mathrm{B_c}$. Let f be Lipschitz in u, with Lipschitz constant l_u , for all $t \geq 0, \mathbf{x} \in \mathrm{B_h}, \mathrm{u} \in \mathrm{B_c}$. Let $\mathrm{u} \in \mathrm{L}_\infty$. If $\mathbf{x} = 0$ is an exponentially stable equilibrium point of the unperturbed system, then: (1) The perturbed system is small-signal L_∞ stable, that is, there exist $\gamma_\infty, c_\infty > 0$, such that $\|u\|_\infty < c_\infty$ implies that $\|\mathbf{x}\|_\infty \leq \gamma_\infty \ \|u\|_\infty < h$, where $\mathbf{x}$ is the solution of $f(t, \mathbf{x}, u)$ starting at $\mathrm{x}_0 = 0$. (2) There exists $m \geq 1$ such that, for all $\|\mathbf{x}_0\| < h/m, 0 < \|u\|_\infty < c_\infty$ implies that $\mathbf{x}$ (t) converges to a B_δ ball of radius δ=γ_∞ $\|u\|_\infty < h$, that is: for all $\varphi > 0$, there exists $T \geq 0$ such that $\|\mathbf{x}(t)\| \leq (1 + \varphi)\delta$ for all $t \geq T$, along the solutions of $f(t, \mathbf{x}, u)$ starting at $\mathbf{X}_0$. Also for $t \geq 0, \|\mathbf{x}(t)\| < h$.

III. GENERAL MODEL DESCRIPTION

In this paper the closed loop identification algorithm considered is applied to a perturbed DC servomechanism with the next mathematical description:

$$J\ddot{q}(t) + f\dot{q}(t) = ku(t) + v_1(t) = \tau(t) + v_1(t) \qquad (3)$$

with J, f, k, u, v_1 and τ being the inertia, viscous friction coefficient, amplifier gain, input voltage, perturbation signal, and torque input, respectively. Model (3) assumes that the amplifier works in current mode. Now (3) is rewritten as:

$$\ddot{q}(t) = a\dot{q}(t) + bu(t) + v(t) \qquad (4)$$

where $a = \frac{f}{J}, b = \frac{k}{J}$ are positive constants and $v = \frac{v_1}{J}$. In all analysis we consider that the perturbation signal is bounded, i.e., $\|v_1(t)\| \leq \beta, \beta \in \mathbb{R}^+$. The objective of the following analysis is to show that when the perturbation signal is identically zero the overall closed loop system, including the parameter error vector, is exponentially stable and that, in the general case, when the perturbation signal is different to zero, the parameter estimates converge to a region Ω_δ which can be arbitrarily reduced by the variation of not only the gains of the matrix involved with the parameter estimation algorithm, but also through the variation of the gains of the controller used for closing the loop of the servomechanism.

IV. CLOSED-LOOP IDENTIFICATION ALGORITHM: NON PERTURBED CASE

The procedure used to perform the closed-loop identification algorithm follows the same lines of that presented in [6]. In this first part we assume that the perturbation signal is identically zero, i.e., $v(t) \equiv 0$.

Fig. 1 shows the closed loop identification procedure which can be described as follows: A PD controller is used for closing the loop of the real servomechanism and a model of it also created, where another PD controller also closes the loop around it, and the same gains k_p and k_d are used for both controller.

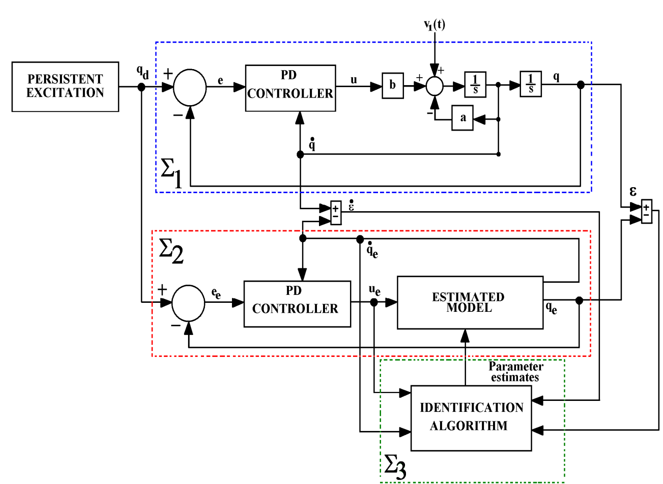

Fig. 1. Blocks diagram for the closed-identification algorithm

IEEE Catalog Number: CFP11827-ART
ISBN: 978-1-4577-1013-1

then, some of the signals of the real closed-loop system and its model, including the output error $\epsilon(t) \triangleq q(t) - q_e(t)$ and its time derivate, are employed to feed an identification algorithm which estimates the system parameters and updates them to the estimated model. All this procedure is now theoretically summarized. The PD controller used for closing the loop around the unperturbed system is given by: $u(t) = k_p e(t) - k_d \dot{q}(t)$, with $e(t) = q_d(t) - q(t)$ being the tracking error signal, $q_d(t)$ the reference signal, and k_p, k_d positive constants. Then, the unperturbed system in closed loop with $u(t)$ leads to the closed-loop dynamics: $\sum_1: \ddot{q}(t) = -(a + bk_d)\dot{q}(t) + bk_p e(t)$. It is not difficult to prove, by using Routh Hurwitz criterion, that the control law $u(t)$ stabilizes $\sum_1$. The estimated model of $\sum_1$ is given by: $\ddot{q}_e(t) = -\hat{a}\dot{q}_e(t) + \hat{b}u_e(t)$. Let consider the PD controller applied to the model: $u_e(t) = K_p e_e(t) - k_d \dot{q}_e(t)$, where $e_e(t) = q_d(t) - q_e(t)$ is the tracking error for the estimated model. Then, the estimated model of $\sum_1$ in closed loop with $u_e(t)$ leads to the closed-loop dynamics: $\sum_2: \ddot{q}_e(t) = -(\hat{a} + \hat{b}k_d)\dot{q}_e(t) + \hat{b}k_p e_e(t)$. As mentioned above, both controllers $u(t)$ and $u_e(t)$ use the same gains. An important issue now is to know if the closed loop dynamics $\sum_2$ is stable because it contains time varying signals. Then, next analysis will let to conclude this topic.

Let output error between the real servomechanism and its model be given as: $\epsilon(t) = (q - q_e)$, then, its possible to evaluate its second time derivate and employing $\sum_1$ and $\sum_2$, it can be established the error dynamics as follows:

$$\ddot{\epsilon}(t) + c\dot{\epsilon} + bk_p \epsilon(t) = \tilde{\theta}^T \phi(t) \tag{5}$$

with $c \triangleq (a + bk_d) > 0$ a positive constant and $\tilde{\theta}(t)$, $\phi(t)$ being the parameter error and the regressor vector, respectly, defined as: $\tilde{\theta} \triangleq \hat{\theta} - \theta = \left[\hat{a} - a, \hat{b} - b\right]^T$, $\phi(t) \triangleq (\dot{q_e}, -u_e)^T$. In order to analyze the behavior of the signals involved in the error dynamics (5), passivity based arguments [2] will be used. To this end, let $\mathbf{x} = [\epsilon, \dot{\epsilon}]^T$ be the state vector and consider the following storage function: $H_1(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T \begin{bmatrix} bk_p & \mu \\ \mu & 1 \end{bmatrix} \mathbf{x} = \mathbf{x}^T M \mathbf{x}$, where μ is a real positive constant. It is easy to show that H_1 will be positive definite if $\mu < \sqrt{bk_p}$. Besides, after some straightforward steps it is possible to conclude that if $\mu < \min\{c/2, 2bk_p/c\}$, then the time derivate of H_1 along the trajectories of (5) yields $\dot{H}_1(\epsilon, \dot{\epsilon}) \le \tilde{\theta}^T \phi(\mu\epsilon + \dot{\epsilon}) - c/2(\mu\epsilon + \dot{\epsilon})^2$, i.e., (5) defines an Output Strictly Passive (OSP) operator $\tilde{\theta}^T \phi \to (\mu\epsilon + \dot{\epsilon})$. Moreover, because of the feedback interconnection of passive subsystems is passive, it is intuitive to consider the parameter error dynamics as the following gradient like parameter updating law: $\sum_3: \dot{\tilde{\theta}} = -\Gamma\phi(\mu\epsilon + \dot{\epsilon})$, with $\Gamma = \Gamma^T \epsilon M^{2x2}$ a constant positive definite matrix. Then, by considering the storage function $H_2(\tilde{\theta}) = \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}/2$ it is easy to conclude that $\sum_3$ defines a passive operator $(\mu\epsilon + \dot{\epsilon})$→- $\tilde{\theta}^T \phi$. Finally, let to consider the feedback interconnection of (5) with $\sum_3$ given by:

$$\sum \begin{cases} \ddot{\epsilon}(t) + c\dot{\epsilon}(t) + bk_p \epsilon(t) = \tilde{\theta}^T \phi(t) \\ \dot{\tilde{\theta}} = -\phi(\mu\epsilon + \dot{\epsilon}) \end{cases} \tag{6}$$

then, by considering the storage function $V_1 = H_1 + H_2$, it is easy to prove that $\sum$ is OSP, thus, $(\mu\epsilon + \dot{\epsilon}) \in L_2$[2]. Let $y(t) = (\mu\epsilon + \dot{\epsilon})$ be the output of the interconnected system (6). Then, it is clear that ϵ is the output of an exponentially stable filter whose input belongs to the L_2 space, therefore $\epsilon(t) \to 0$ as $t \to \infty$ [10]. Thus, $q(t) \in L_\infty$ and $\epsilon(t) \to 0$ as $t \to \infty$, therefore, $q_e \to q(t)$ as $t \to \infty$, i.e., $q_e(t) \in L_\infty$ therefore, , it proves the stability of the estimated model $\sum_2$ as desired. However, it has not been proved yet the convergence of the parameter error. To this end let to consider the state-space description of (5) as $\dot{\mathbf{x}} = A\mathbf{x} + Bu$, $y(t)=C\mathbf{x}$, with $A = \begin{bmatrix} 0 & 1 \\ -bk_p & -c \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C^T = \begin{bmatrix} \mu \\ 1 \end{bmatrix}$, $U = \tilde{\theta}^T \phi$, where $\mathbf{x}^T = [\epsilon, \dot{\epsilon}]$ is the state vector. Parameter convergence is ensured if the regressor vector $\phi(t)$ is Persistently Exciting (PE) [11], but it contains time varying signals from the estimated model $\sum_2$, which makes difficult to analyze it and conclude that $\phi(t)$ is PE. To overcome this drawback let to consider the regresor vector $\phi_r^T(t) = [\dot{q}, -u]$ which considering the same lines as [11] it is not a difficult task to show that if $c > \mu$, then, $\phi_r(t)$ will be PE (see also [6]). Now, let to consider the difference $\phi_r - \phi$. The storage function V_1 is positive definite if $\mu < \sqrt{bk_p}$ and after some straightforward calculations it is possible to show that $\dot{V}_1 \le -\beta\epsilon^2, \beta \triangleq \frac{2}{\alpha}\left(\frac{\mu bk_p \alpha}{2} - \frac{\mu^2 c^2}{8}\right)$, if $\alpha \triangleq (c - \mu) > 0$. Therefore, if $\mu \le 4bk_p c/(4bk_p + c^2)$, then, $\epsilon(t) \in L_2$ and it was proven that $y(t) = (\mu\epsilon + \dot{\epsilon}) \in L_2$, thus, $\dot{\epsilon}(t) \in L_2$, which makes clear that $(\phi_r - \phi) \in L_2$. It can also be shown [11] that given a PE signal $\omega(t)$ and a signal $z(t) \in L_2$, the sum $(\omega + z)$ is still PE [11]. Thus, $\phi = \phi_r - (\phi_r - \phi)$ is PE as desired and the convergence of $\tilde{\theta}(t)$ to zero can be claimed, i.e., the estimated parameters do converge to the real ones. Based on the above results, the following proposition follows.

Proposition 14: Let the system $\sum_1$ and assume that $\mu < \min\{\sqrt{bk_p}, c/2, 2bk_p/c, 4bk_p c/(4bk_p + c^2)\}$. Then

$$\{\epsilon(t), \dot{\epsilon}(t), \tilde{\theta}(t)\}_{t\to\infty} \to 0 \tag{7}$$

The results presented above show the asymptotic stability of all the signals involved in the identification algorithm, including the convergence of the parameter error to zero. However, in order to make the algorithm robust when there exist disturbances, it is important to prove not

IEEE Catalog Number: CFP11827-ART
ISBN: 978-1-4577-1013-1

only asymptotic stability, but also exponential stability, which is the next topic.

A. Exponential Convergence

In system identification, the exponential convergence is an important feature to make the identification algorithm robust in face of perturbation signals. Thus, it is important to show under which conditions the system (6) is indeed exponentially stable. Let consider the state space description of (6), then the system (A,B,C) is observable and controllable if $\mu < c/2$, i.e., it is a minimal realization. Let consider the systems:

$$\Sigma_4 : \begin{cases} \dot{\tilde{\theta}} = 0 \\ y_2 = \phi^T(t)\tilde{\theta}(t) \end{cases}, \Sigma_5 : \begin{cases} \dot{\tilde{\theta}} = -\Gamma\phi(\mu\epsilon + \dot{\epsilon}) \\ y_2 = \phi^T(t)\tilde{\theta}(t) \end{cases} \tag{8}$$

The analysis of the last section proved that $\phi(t)$ is PE, therefore $\sum_4$ is UCO and so do $\sum_5$ (see Definition 1 and Lemma 2). Then, the main result of this section can be established as follows and follows the same lines presented in [13].

Theorem 15: Let the state space description of (5) and assume that the following inequalities hold: $\mu < \min \sqrt{bk_p}, \frac{1}{2c}, \frac{2bk_p}{c}, \frac{4bk_pc}{4bk_p+c^2}$. Let $\bar{w} = (\bar{\mathbf{x}}, \bar{\tilde{\theta}})$ be an equilibrium point of the system. Then $\bar{\mathbf{w}}$ is an exponentially stable equilibrium point.

Proof: See [13] for details of the proof.

Now it has been proved the exponential stability for the state space description of (6) in the non perturbed case. However, in practical applications it is well known that there will be some disturbances that will appear, such as unmodelled dynamics or noise from the environment or the measuring devices, which makes necessary to extend the analysis to the perturbed case, as described in the next Section.

V. CLOSED-LOOP IDENTIFICATION: PERTURBED CASE

The closed-loop identification of the perturbed servomechanism will be considered in this Section. The model under study is given by (4). Let to consider the system (4) in closed loop with $u(t)$ and the estimated model Σ_2 in closed loop with $u_e(t)$, and the output error $\epsilon(t)$ defined as before. Then, it is possible to obtain the following error dynamics:

$$\dot{\epsilon}(t) = -c\dot{\epsilon}(t) - bk_p\epsilon(t) + \theta^T\phi(t) + v(t) \tag{9}$$

Stability of (9) will be considered and if this property can be stated, then, the system will be robust even in the presence of perturbations signals, which is desirable in the identification context because reliable parameter estimates can be obtained even in the perturbed case. The next main result let us conclude that the perturbed system (4) is L_∞ stable.

Theorem 16: Consider the perturbed system (9) and the unperturbed system (5). If the equilibrium $\mathbf{w}_0$ of (5) is exponentially stable, then: (1) The perturbed system (9) is small signal L_∞-stable, that is, there exist γ_∞ such that: $\|\mathbf{w}(t)\| \leq \gamma_\infty\beta < h$, where $\mathbf{w}(t)$ is the solution of (9) starting at $\mathbf{w_0}$. (2) There exists $m \geq 1$, such that, $\|\mathbf{w}_0\| < h/m$ implies that $\mathbf{w}(t)$ converges to a Ω_δ ball of radius $\delta = \gamma_\infty\beta < h$, that is: for all $\epsilon > 0$, there exists $T \geq 0$ such that $\|\mathbf{w}(t)\| \leq \delta(1+\epsilon)$ for all $t \geq T$, along the solutions of (9) starting at $\mathbf{w_0}$. Also for all $t \geq 0$, $\|\mathbf{w_0}\| < h$.

Proof: See [13] for details of the proof.

VI. ANALYSIS OF THE RESULTS

From the stability proof of the perturbed system (9) it is possible to obtain the bound (see [13]): $\gamma_\infty\beta = \frac{\sqrt{u^2+1}}{\lambda_{min}(Q)}\sqrt{\frac{\max(\lambda_{max}(M_2),\lambda_{max}(\Gamma))}{\min(\lambda_{min}(M_2),\lambda_{min}(\Gamma))}}\beta$. The eigenvalues for M_2 are given by: $s = \frac{(b_{kp}+\mu c+1)\pm\sqrt{(bk_p+\mu c-1)^2+4\mu^2}}{2}$, so that:

$$\lambda_{min}(M_2) = \frac{(b_{kp} + \mu c + 1) - \sqrt{(bk_p + \mu c - 1)^2 + 4\mu^2}}{2}$$

$$\lambda_{max}(M_2) = \frac{(b_{kp} + \mu c + 1) + \sqrt{(bk_p + \mu c - 1)^2 + 4\mu^2}}{2}$$

$$\lambda_{min}(Q) = \min\{\mu bk_p, c - \mu\}$$

$$\lambda_{max}(Q) = \max\{\mu bk_p, c - \mu\}$$

In order to make the region Ω_δ as small as possible we need to minimize the value of $\lambda_{max}(M_2)$ and maximize that of $\lambda_{min}(M_2)$, i.e., the value of $\sqrt{(bk_p + \mu c - 1)^2 + 4\mu^2}$ must be the minimum possible. To investigate the conditions needed to do this, let to consider the function: $f_m(\mu, k_p, k_d) = (b_{kp} + \mu c + 1)^2 + 4\mu^2$,where: $\frac{\partial f}{\partial k_p} = 2b(bk_p + \mu c - 1), \frac{\partial^2 f}{\partial k_p{}^2} = 2b^2$, $\frac{\partial f}{\partial k_d} = 2\mu b(bk_p + \mu a + \mu bk_d - 1), \frac{\partial^2 f}{\partial k_d{}^2} = 2\mu^2 b^2$, $\frac{\partial f}{\partial \mu} = 2\mu(bk_p + \mu c - 1) + 8\mu, \frac{\partial^2 f}{\partial \mu^2} = 4\mu c + 2bk_p + 6$, and the respective equilibrium points $\{k_p^*, k_d^*, \mu^*\}$ are given by: $k_p^* = \frac{1-\mu(a+bk_d)}{b}$, $k_d^* =$

IEEE Catalog Number: CFP11827-ART
ISBN: 978-1-4577-1013-1

$\frac{1-(bk_p+\mu a)}{\mu b}$ and $\mu^* = \frac{3+bk_p}{a-bk_d}$. Therefore, from the partial derivatives of f_m it is clear that the equilibrium points correspond to a minimum. Now it is possible to analyze the qualitative behavior of the region Ω_δ as follows: (a) For a given constant value of k_d and μ, if k_d is big, the value of k_p^* is reduced, then, the size of the region Ω_δ is reduced. (b) For a given constant value of k_p, if k_p is big, the values of k_d^* and μ are reduced, then the size of the region Ω_δ is reduced. (c) The region Ω_δ can be made arbitrarily small if the values for $\lambda_{min}(Q)$ and $\min(\lambda_{min}(M_2), \lambda_{min}(\Gamma))$ are as large as possible. (e) The value for $\lambda_{min}(Q)$ can be increased if the controller gains k_d and k_p have large values, which is the case if we use a high gain controller. (f) By increasing the values of the gain matrix Γ we increase the value of $\lambda_{min}(\Gamma)$, which tends to reduce the region Ω_δ. Figure 2 shows the behavior of the region Ω_δ. In all the cases it can be seen that if we increase the value of gains of the matrix Γ, kp, k_d and μ, the size of the region Ω_δ is reduced as expected, making clear that the controller gains are also important if we want to reduce the region of convergence, which shows that the parameter estimates are reliable even in the presence of disturbances.

From the last analysis or is possible to conclude that for robustness of the identification algorithm, it is important to pay attention not only to the values of the gain matrix Γ and μ, but also to the value of the controller gains k_p and k_d. Region Ω_δ is effectively reduced if high gain k_p is employed, and also if large eigenvalues are obtained from matrix Γ. Another subject to be considered is the controller structure, because it is possible to note that not all the gains do have the same effect of reducing region Ω_δ. Therefore, when the parameter identification algorithm is designed, it is important to note that the structure of the controller should be an important issue that has been underestimated and that will be important in real applications, where it is impossible to have a system without any noisy measurement or unmodeled dynamics affecting it.

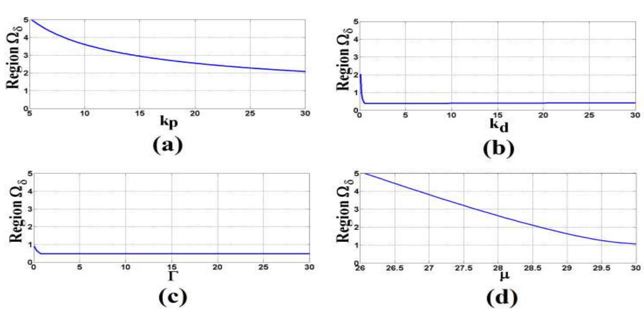

Fig. 2. (a) Behavior of the size of region Ω_δ when the value of the gain k_p is increased. (b) Behavior of the size of region Ω_δ when the value of the gain k_d is increased. (c) Behavior of the size of region Ω_δ when the value of the gains from matrix Γ are increased. (d) Behavior of the size of region Ω_δ when the values of gains from matrix μ are increased

VII. CONCLUSION

This paper presents a method for on-line closed loop identification of a linear DC servomechanism working in closed loop with a bounded perturbation signal. A PD controller closes the loop around of both the real and an estimated model. An advantage of the configuration used to perform closed loop identification is that it allows freely choosing the excitation signal. It is proved that in absence of a perturbation signal the system is exponentially stable, and in the perturbed case the system remains being stable, making it robust in face of perturbation signals. Finally, it was shown that by increasing the controller and adaptation gains the region Ω_d, where the parameter estimates converge to, can be reduced, which means that in this way the parameters obtained are reliable, giving some insight about the importance of the structure of the controller in the identification process. Future work will be devoted to design more complicated controllers and verify which controller structure will lead to a better estimate, i.e., a controller which reduces Ω_δ as much as possible and ensures the overall exponential stability.

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IEEE Catalog Number: CFP11827-ART
ISBN: 978-1-4577-1013-1