Parameter Identification of a Perturbed Servomechanism Operating in Closed Loop under PID control

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Abstract — **This paper presents a methodology for closed-loop identification of a position-controlled servomechanism subject to a constant disturbance. A Proportional Integral Derivative controller, which stabilizes the servomechanism without knowledge about its parameters, closes the loop. Theoretical results show that the integral action plays a fundamental role in counteracting the effect of the disturbance on the parameter estimates. Experiments on a laboratory prototype show that the estimates obtained under disturbance conditions remain closer to that obtained under disturbance-free conditions.**

Keywords — Parameter identification, persistent excitation, DC servomechanism, closed-loop identification.

I. Introduction

Servomechanisms play a fundamental role in modern Robotic and Mechatronic systems where high speed and high precision are of prime importance. In most industrial controllers, Proportional Integral Derivative (PID) algorithms are the choice for closing the loop when the variable of interest is the servomechanism position [1]. Even if a PID controller tuned manually would give acceptable results, model-based tuning procedures could improve closed loop performance. A necessary step for applying most of the model-based tuning methods is to perform a parameter identification procedure on the servomechanism. In this regard, it is worth noting that if the variable of interest is position, then, a linear model of a servomechanism contains a pole on the imaginary axis. The above means that a bounded input applied to the servomechanism would not produce a bounded position. This behavior indicates that, for obvious security reasons, parameter identification should be performed in closed loop. Unfortunately, most of the methods found in the literature [2], [3], assume a stable behavior of the plant under identification and cannot be directly applied for identifying a servomechanism devoted to position control.

References [4], [5], [6], [7] propose methods for closed loop identification of position-controlled servos where the loop is closed using a linear controller. In [4] the authors performs parameter identification using a low gain proportional controller combined with an off-line Least Squares identification algorithm. In the method presented in [5], a two degrees-of-freedom linear controller closes the loop; the controller is tuned using the rotor inertia and setting the viscous and Coulomb friction to zero. Subsequently, an off-line Least Squares algorithm allows estimating these parameters. Next, the parameter estimates obtained previously allows computing again the controller. In [6] a recursive Multi-Step Recursive Extended Least Squares permits identifying a linear discrete-time model of a servo. The voltage input and the position of the servomechanism feed the estimation algorithm and a low gain proportional controller closes the loop. The approach proposed in [7] uses a PD controller to close the loop and an on-line gradient algorithm allows estimating a linear model of a servomechanism. Relay-based techniques are widespread for servo identification [8], [9]. The idea behind these methods, which is similar to the relay tuning methods in process control [10], is to close the loop through a relay in order to obtain a sustained oscillation. Then, its amplitude and frequency allows identifying linear model and non linear models of a servomechanism.

The review previously presented allows making the following observations. Nearly all of the identification procedures using a linear controller [4], [5], [6] fall into the category of direct methods [11], i.e., the parameter identification procedure is applied without regard about the controller being used to close the loop. The method proposed in [7] corresponds to an indirect method, i.e., the controller is taken into account into the parameter identification method. Moreover, most of them use Least Squares methods, which would give incorrect estimates if the disturbances affecting the servo have not zero mean as in the case of constant load torques or parasitic constant voltages in the amplifier driving the servomotor. On the other hand, the relay-based methods give consistent results but their drawback is the fact that tuning of the relay controller can be cumbersome and some of these methods require prior knowledge on the servomechanism parameters. Moreover, no one of the reviewed methods takes explicitly into account disturbances affecting the servomechanism.

This work presents an identification methodology for a position-controlled servomechanism. A PID controller closes the loop and achieves stability without knowledge about the servomechanism parameters. Theoretical results show that the integral action counteracts the effect of a constant disturbance on the parameter estimates; moreover, experiments in a laboratory prototype show that parameter estimates obtained under a constant disturbance remain closer to the estimates obtained when no disturbances affect the servomechanism. The paper layout is as follows. Section 2 presents the proposed identification methodology and Section 3 exposes its parameters convergence properties. Section 4 shows experimental results sustaining the

proposed approach. Finally, the paper ends with some concluding remarks.

II. PARAMETER IDENTIFICATION ALGORITHM

Consider a servomechanism composed of a servomotor, a servo-amplifier and a position transducer; a linear filter provides velocity estimates from position measurements. The following is a mathematical model of a perturbed servomechanism

$$J\ddot{q} = -f\dot{q} + ku + p \tag{1}$$

where q, $\dot{q}$ and $\ddot{q}$ are respectively its position, velocity, and acceleration; u is the voltage input, J and f are the servomechanism inertia and viscous friction; k is the input gain, which is a function of the gain of the amplifier driving the servomotor; and p a constant disturbance. This model is valid for DC brushed, and DC and AC brushless servomotors if the servo-amplifier driving the servomotor works in current mode. Rewriting the servomechanism model yields

$$\ddot{q} = -a\dot{q} + bu + d \tag{2}$$

The parameters of the model are $a = f/J$, $b = k/J$, and $d = p/J$. The following paragraphs describe the proposed methodology (see Fig. 01). Consider the PID control law

$$u = k_p e - k_d\dot{q} + k_i \int_0^t e(\tau)d\tau \tag{3}$$

closing the loop around the servomechanism with k_p, k_d, and k_i positive constants. The expression $e = q_d - q$ corresponds to the position error and q_d is a reference signal. Substituting (3) into (2) leads to the following closed-loop system

$$\ddot{q} = -\alpha\dot{q} + bk_p e + bk_i z \tag{4}$$

with $z := (\int_0^t e(\tau)d\tau + d/(bk_i))$ and $\alpha := a + bk_d$. It is not difficult to show that the closed loop system (4) is stable if $k_p > k_i/\alpha$ holds. Now, consider a model of the servomechanism described as

$$\ddot{q}_e = -\hat{a}\dot{q}_e + \hat{b}u_e \tag{5}$$

with $\hat{a}$ and $\hat{b}$ being the estimates of a and b, respectively. The following PID control law closes the loop around (5)

$$u_e = k_p e_e - k_d\dot{q}_e + k_i \int_0^t e_e(\tau)d\tau \tag{6}$$

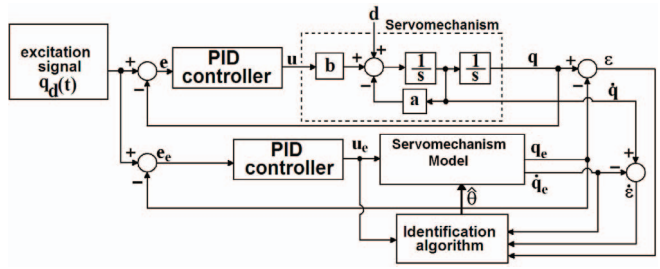

Fig. 01. Block diagram of the proposed closed-loop identification algorithm.

where the variable $e_e := (q_d - q_e)$ defines the estimated output error. Note that (3) and (6) share the same structure and the gains in both controllers are the same. Substituting (6) into (5) leads to

$$\ddot{q}_e = -\hat{\alpha}\dot{q}_e + \hat{b}k_p e_e + \hat{b}k_i z_e \tag{7}$$

with $\hat{a}$ and $\hat{b}$ being time-varying estimates of a and b respectively provided by an identification algorithm and $z_e = \int_0^t e_e(\tau)d\tau,\ \hat{\alpha} := \hat{a} + \hat{b}k_d$.

A. Stability Analysis

Two aspects about the proposed method require further study. Firstly, even if the closed-loop system (4) is stable, the same conclusion cannot be drawn for (7) due to its time-varying nature. Secondly, at this stage there is not an explicit expression for the identification algorithm. The aim of the analysis described in the next paragraphs is to respond to these queries. Define the error between the output of the servomechanism and its model as $\epsilon := q - q_e$. Taking the second-time derivative of ϵ and employing (4) and (7) yields

$$\ddot{\epsilon} = -\alpha\dot{\epsilon} - bk_p\epsilon - bk_i\omega + \tilde{\theta}^T\phi \tag{8}$$

where the parameter error vector $\tilde{\theta}$, regressor vector ϕ and the variable ω are defined as

$$\tilde{\theta} := \hat{\theta} - \theta = \begin{bmatrix} \hat{a} - a \\ \hat{b} - b \end{bmatrix}, \phi := \begin{pmatrix} \dot{q}_e \\ k_d\dot{q}_e - k_p e_e - k_i z_e \end{pmatrix} \tag{9}$$
$$\omega := z_e - z = \int_0^t \epsilon(\tau)d\tau - \frac{d}{bk_i}$$

Now, Passivity arguments [5] will allow performing the stability analysis for the error dynamics (8). Let consider the storage function

$$H_1 = \frac{1}{2}\begin{bmatrix} \epsilon \\ \dot{\epsilon} \\ \omega \end{bmatrix}^T \begin{bmatrix} bk_p & \mu & bk_i \\ \mu & 1 & 0 \\ bk_i & 0 & \mu bk_i \end{bmatrix} \begin{bmatrix} \epsilon \\ \dot{\epsilon} \\ \omega \end{bmatrix} \tag{10}$$

with $\mu > 0$. Note that (10) is positive definite if $k_p > (\mu^3 + bk_i)/(\mu b)$. Taking the time derivative of (10) along the trajectories of (8) gives

$$\dot{H}_1 \leq \tilde{\theta}^T \phi(\mu\epsilon + \dot{\epsilon}) - \frac{\alpha}{2}(\mu\epsilon + \dot{\epsilon})^2$$

where the inequalities $\alpha > \mu$, $k_p > (2bk_i + \alpha\mu^2)/(2\mu b)$ were considered to be valid. Therefore, (8) describes an Output Strictly Passive (OSP) mapping, i.e., $\tilde{\theta}^T\phi \to (\mu\epsilon + \dot{\epsilon})$ with input $v_1 := \tilde{\theta}^T\phi$, output

$y_1 := (\mu\epsilon + \dot{\epsilon})$, and L_2-gain $\gamma = \alpha/2$. Now, consider the

following update law

$$\dot{\hat{\theta}} = \dot{\tilde{\theta}} = -\Gamma\phi(\mu\epsilon + \dot{\epsilon}) \qquad (11)$$

where $\Gamma \in \mathbb{R}^{2\times 2}$, $\Gamma = \Gamma^T > 0$ and let consider the storage function $H_2(\tilde{\theta})$ described as

$$H_2(\tilde{\theta}) = \frac{1}{2}\tilde{\theta}^T\Gamma^{-}1\tilde{\theta} \qquad (12)$$

The time derivative of H_2 along the trajectories of (11) shows that (11) defines a passive operator $(\mu\epsilon + \dot{\epsilon}) \to -\tilde{\theta}^T\phi$ with input y_1 and output $-v_1$. Therefore, the negative feedback interconnection

$$\begin{cases} \ddot{\epsilon} = -\alpha\dot{\epsilon} - bk_p\epsilon - bk_i\omega + \tilde{\theta}^T\phi \\ \dot{\hat{\theta}} = \dot{\tilde{\theta}} = -\Gamma\phi(\mu\epsilon + \dot{\epsilon}) \end{cases} \qquad (13)$$

is an OSP mapping with $y_1 \in L_2$. Besides, note that ϵ is the output of a linear first order exponentially stable system whose input belongs to the L_2 space; consequently [13] $\epsilon \xrightarrow[t\to\infty]{} 0$. Now, let consider the Lyapunov function candidate $V_1 = (H_1 + H_2)$. It is easy to show that the time derivative of V_1 along the trajectories of (13), assuming that $k_p > (\mu^3 + bk_i)/(\mu b)$, $\alpha/2 > \mu$ and $k_p > (2bk_i + \alpha\mu^2)/(2\mu b)$ hold, is $V1 \leq -\frac{\alpha}{2}(\mu\epsilon + \dot{\epsilon})^2$. Hence, $\{\epsilon, \dot{\epsilon}, \omega, \tilde{\theta}\}$ are bounded and so do $\{q_e, \dot{q}_e, z_e, \phi, \ddot{\epsilon}\}$. Since $\omega \in L_\infty$ and $\dot{\omega}(t) = \epsilon \in L_\infty$, then ω is uniformly continuous [13] and $\omega \xrightarrow[t\to\infty]{} 0$. From the above, it is clear that $y_1 \in L_\infty$ and $\dot{y_1} \in L_\infty$. Therefore, y_1 is uniformly continuous and $y_1 \xrightarrow[t\to\infty]{} 0$ [13]. As a consequence $\dot{\epsilon} \xrightarrow[t\to\infty]{} 0$. The above result follows in view of the fact that $\dot{\epsilon} = y_1 - \mu\epsilon$.

The precedent analysis shows that the signals $\{q_e, \dot{q}_e, z_e, \tilde{\theta}\}$ remain bounded and $\{\epsilon, \dot{\epsilon}, \omega\} \xrightarrow[t\to\infty]{} 0$; therefore, the closed loop system (7) is stable. Furthermore, the update law (11) estimates the servomechanism parameter θ. All the above inequalities can be are summarized as

$$k_p > max\{k_i\alpha^{-1}, (2bk_i + \alpha\mu^2)(2\mu b)^{-1}\} \qquad (14)$$
$$k_i > 0, k_d > 0, \tfrac{1}{2}\alpha > \mu > 0$$

The next proposition resumes the previous analysis.

Proposition 1. *Consider the servomechanism (2) in closed loop with the PID control law (3) and the servomechanism model (5) in closed loop with the PID control law (6). If the identification algorithm (11) updates the parameter estimates of the servomechanism model and (14) holds, then, (4) is stable, all the signals in (7) remain bounded and*

$$\{\epsilon, \dot{\epsilon}, \omega\} \xrightarrow[t\to\infty]{} 0$$

III. PARAMETER CONVERGENCE

A well-known result in Parameter Identification is that the parameter error $\tilde{\theta}(t)$ converges exponentially to zero if the regressor vector $\phi(t)$ is Persistently Exciting (PE). The following analysis establishes the conditions under which this property holds. The analysis is divided into three steps. Fisrt, establishing the PE condition of the regressor vector related to the servomechanism (2) when $d = 0$. Second, determining the PE condition of the regressor vector associated to the servomechanism (2) when $d \neq 0$. Third, verifying the PE condition of the regressor vector corresponding to the model (5).

A. PE condition of the servomechanism without disturbance

Let consider the servomechanism (4) without perturbation, i.e., $d = 0$

$$\ddot{q} = -\alpha\dot{q} + bk_pe + bk_i\bar{z} \qquad (15)$$

where $\bar{z} := \int_0^t e(\tau)d\tau$. Define the vector ϕ_1 corresponding to (15) as $\phi_1 := [\dot{q}, k_d\dot{q} - k_pe - k_i\bar{z}]^T$. Using arguments similar to those presented in [14], it is possible to show that if the reference $q_d(t)$ is sufficiently rich (SR) of order n, then, ϕ_1 is PE.

B. PE condition of the servomechanism under disturbances

The following result relates the PE condition of two regressor vectors.

Lemma 1. *([15]) The regressor vector $\psi(x)$ is PE with level $\delta_0 > 0$ if there exist constants $\{t_1, \epsilon_0, \delta_0\} > 0$ such that $\| \psi(x(t),t) - \psi(x_d(t),t) \| < \frac{\epsilon_0 - \delta_0}{T}$ for all $t \geq t_1$ and $\| \mathbf{e}(t) \| \leq \epsilon_1$, where $\mathbf{e}(t) = [e_1(t), e_2(t)]^T = x(t) - x_d(t)$ and $\psi(x_d(t),t)$ is a vector fulfilling the PE conditions with level ϵ_0, i.e.,*

$$\int_{t'}^{t'+T} | \zeta^T \psi(x_d(t),t) | \, dt \geq \epsilon_0 \qquad (16)$$

where ζ is an arbitrary unit vector and $t' > 0$.

In order to apply Lemma 1 consider the vector $\phi_2(t)$ defined as $\phi_2(t) := [\dot{q}(t), k_d\dot{q} - k_p e - k_i z]^T$ which corresponds to the perturbed servomechanism (2), and $\| \phi_1 - \phi_2 \| = d/b$. Let ϵ_0 be the excitation level of ϕ_1 and $(\epsilon_0 - \delta_0)/T = d/b$. If the disturbance d is small enough, then ϕ_2 is PE with excitation level $\delta_0 = \epsilon_0 - Td/b$.

A. PE condition for the servomechanism model

The stability analysis of Section 3 shown that $\dot{q}_e(t) \to q(t), q_e(t) \to q(t), z_e(t) \to z(t)$. Moreover, $e_e(t) \to e(t)$ because $(e_e - e) = \epsilon \to 0$. Therefore, $\phi(t) \xrightarrow[t\to\infty]{} \phi_2(t)$. As a consequence, ϕ_2 fulfills asymptotically a PE condition. It is worth remarking that this results holds in spite of a disturbance affecting the servomechanism.

II. EXPERIMENTAL RESULTS

The servomechanism employed for the experiments consists of a DC brushed Clifton Precision motor, model JDTH-2250-BQ-IC, driven by a Copley Controls analog power servoamplifier, model 413, configured in current mode. An optical encoder gives angular position measurements. A MultiQ-3 card from Quanser Consulting performs data acquisition. The Matlab/Simulink software operating with the WINCON software from Quanser Consulting serves as programming platform. Fig. 02 shows the experimental setup. The sampling period for all the experiments is set to 1 *ms*. The MatLab/Simulink block Band Limited White Noise provides the signal excitation and its parameters are set to Noise Power = *0.005*, seed [1212121] and Sampling time = *0.1 s*.

The parameters of model (2), which in the sequel are called the Nominal Parameters, were computed using the technical data of the servomotor and the power amplifier. The values are *a = 0.2174* and *b = 257.7*. For adding a constant disturbance *d* to the servomechanism, the power amplifier is left unbalanced. The experiments consider two cases depending on whether or not a disturbance exists. Fisrt, the servoamplifier is balanced and second when the servoamplifier is unbalanced. The gains for the PID controller are $k_p = 9, k_d = 0.19, k_i = 4$. The update law gains are set to $\Gamma = diag\{0.2, 110\}$ and $\mu = 10$. Fig. 03-(1) and Fig. 03-(2) show the time evolution of the parameter

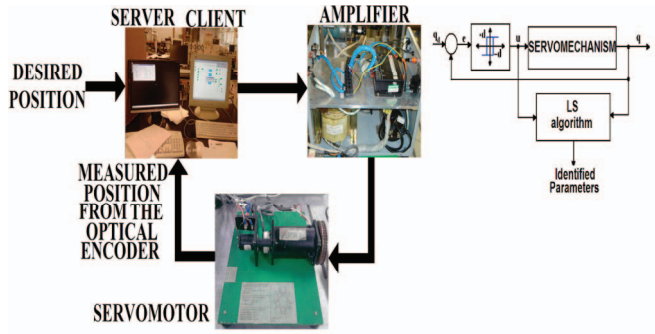

Fig. 02. Programming platform (left) and Least Squares block's diagram.

estimates under no disturbance conditions with the proposed approach. Fig. 03-(5) and Fig. 03-(6) depict the time evolution of the parameter estimates with the proposed approach when there exists a constant disturbance. Note that the parameter estimates found using the proposed method, are close to those computed using the servomechanism technical data. Moreover, these results show that the proposed approach is robust in face of constant disturbances. Performance of the proposed approach was compared against a recursive discrete-time Least Squares algorithm with forgetting factor [2]. In this case, a relay closes the loop (see Fig. 02) and the regression model was obtained in the same way as in [14] by filtering the servomechanism input and ouput. Fig. 03-(3) and Fig. 03-(4) show the time evolution of the parameter estimates under no disturbance conditions with the LS algorithm. Note that in this case the estimates are close to the servomechanism nominal values. Fig. 03-(7) and Fig. 03-(8) correspond to the case when the amplifier is left unbalanced with the LS algorithm; the parameter estimates are different from the estimates when the amplifier is balanced.

II. CONCLUSION

This paper exposes a method for on-line identification of the parameters of a linear model of a servomechanism working in closed loop. A PID controller stabilizes the servomechanism and makes the identification procedure robust in face of constant disturbances. An advantage of this configuration is that it allows freely choosing the excitation signal. Experiments conducted on a laboratory prototype allow evaluating the performance of the proposed approach. The proposed algorithm produces estimates similar to those computed from the technical data of the servomechanism. A Least Squares algorithm produces estimates under disturbance conditions, which are different from the estimates when there are not disturbances affecting the servomechanism.

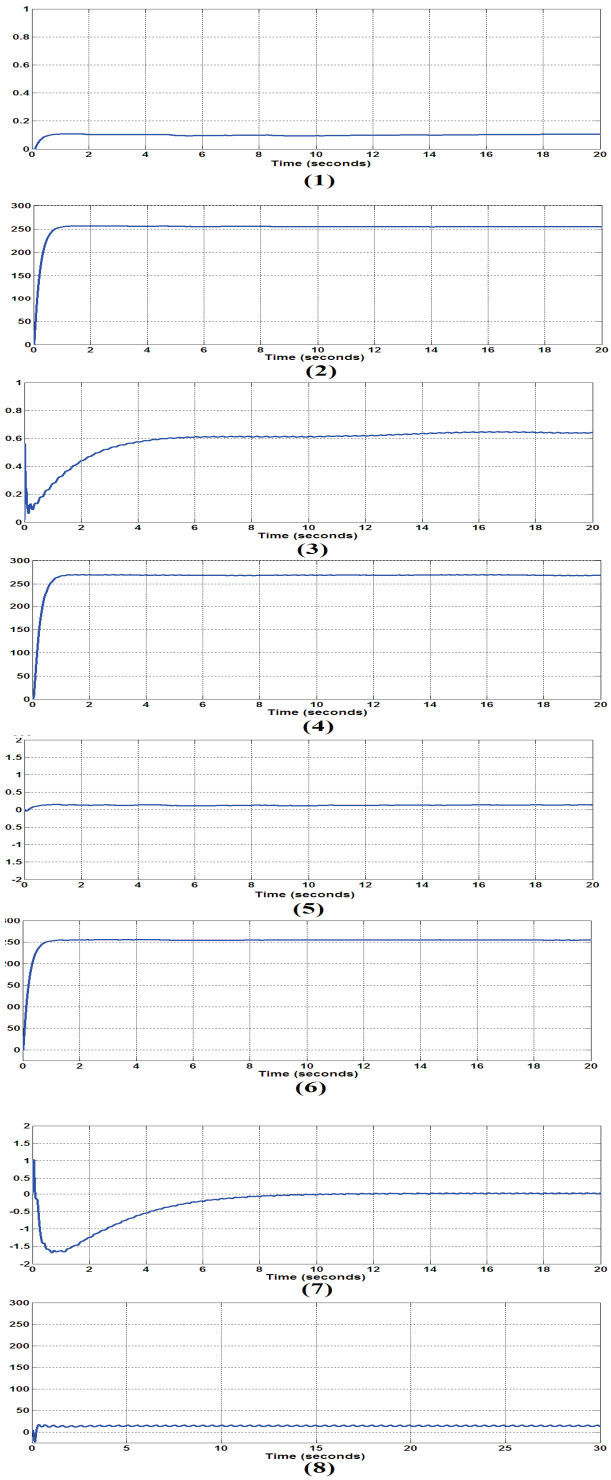

Fig. 03. Identified parameters: (1) Parameter $\hat{a}$ obtained using the proposed method: $d = 0$, (2) Parameter $\hat{b}$ obtained using the proposed method: $d = 0$, (3) Parameter $\hat{a}$ obtained using the LS method: $d = 0$, (4) Parameter $\hat{b}$ obtained using the LS method: $d = 0$, (5) Parameter $\hat{a}$ obtained using the proposed method: $d \neq 0$, (6) Parameter $\hat{b}$ obtained using the proposed method: $d \neq 0$, (7) Parameter $\hat{a}$ obtained using the LS method: $d \neq 0$, (8) Parameter $\hat{b}$ obtained using the proposed method: $d \neq 0$.

References:

[1] G. Ellis. Control systems design guide. Second Edition. Academic Press, 2000.

[2] Nelles, Oliver, "Nonlinear system identification", Ed. Springer Verlag, 2001

[3] L. Ljung, "System Identification", Ed. Prentice Hall, 1987

[4] E. J. Adam, E.D. Guestrin, "Identification and robust control for an experimental servomotor", ISA Transactions, vol. 41, no. 2, pp. 225-234, 2002.

[5] T. Iwasaki, T. Sato, A. Morita, M. Maruyama, "Autotuning of two degree of freedom motor control for high accuracy trajectory motion", Control Eng. Practice, vol. 4, no. 4, pp.537-544, 1996.

[6] Y. Zhou, A. Han, S. Yan, X. Chen, "A fast method for online closed loop system identification", The International Journal of Advanced Manufacturing Technology, vol. 31, no. 1-2, pp. 78-84, 2006.

[7] Garrido, R., R, Miranda. Autotuning of a DC Servomechanism using Closed Loop Identification. IEEE 32nd Annual Conference on Industrial Electronics, IECON 2006. Paris, France, October 7-10, 2006.

[8] K. K. Tan, T.H. Lee, S.N. Huang, X. Jiang, "Friction modeling and adaptive compensation using a relay feedback approach", IEEE Trans. on Industrial Electronics, vol. 48, no.1, pp. 169-176,2001.

[9] A. Besançon-Voda, G. Besançon, "Analysis of a two-relay system configuration with application to Coulomb friction identification", Automatica, vol. 35, no. 8, pp. 1391-1399,1999.

[10] K. J. Åström, T. Hagglund, PID Controllers: Theory, Design and Tuning, 2nd ed., International Society for Measurement and Control, 1994

[11] U. Forsell, L. Ljung, "Closed loop identification revisited", Automatica, vol. 35, no. 7, pp. 1215-1241, 1999.

[12] Ortega, R., Loria, A., Nicklasson, P. J., R., H. Sira, "Passivity-Based Control of Euler-Lagrange Systems", Springer Verlag, London, Ltd., 1998.

[13] Desoer, C. A., Vidyasagar, M., "Feedback Systems: Input-Output Properties", Academic Press, New York, 1975

[14] Sastry, S., Bodson, M., "Adaptive Control: Stability, Convergence and Robustness", Englewood Cliffs, NJ: Prentice Hall, 1989

[15] Huang, J. T., "Parametric identification for Second-Order nonlinear systems in closed-loop operations", Journal of Dynamic Systems, Measurement and Control, Vol. 128, pp. 686-690