Closed-Loop Identification of a Nonlinear Servomechanism: Theory and Experiments

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Abstract. This paper presents a new methodology for parameter identification of a class of nonlinear servomechanisms. The key element is a closed-loop identification technique where a Proportional Derivative controller stabilizes the servomechanism. Experiments using a laboratory prototype, allows comparing the proposed approach against a standard Least Squares algorithm. It is shown that the disturbances acting on the servomechanism do not significantly affect the parameter estimates obtained using the proposed approach.

Keywords: Nonlinear identification, closed-loop identification, servomechanism, persistent excitation.

1 Introduction

Nonlinear servomechanisms typically appear in applications where a brushed or brushless servomotor drives nonlinear loads such as robot arms. However, the problem of estimating the system parameters may get complicated when the plant to be identified must work in closed-loop or if the system is open loop unstable. Identification of systems operating in closed-loop has been a topic of research in the last years because there exist cases where it is not possible to apply directly open-loop identification methods. For example, in unstable industrial processes operating under feedback control, experimental data can only be collected under closed-loop conditions. This last situation remains valid for most mechanical servo systems such as robotic manipulators and high-precision position control systems where the loop may not be removed for security reasons. Several methods for closed-loop identification of linear plants has been proposed in the literature, [1], [2], [3]. However, the presence of nonlinearities in many real-life plants motivates the development of methods suited for this case. References [4] and [5] deal the problem of closed-loop identification of nonlinear time-varying plants operating under linear or nonlinear feedback. In these references the closed-loop system is transformed into one represented using the Youla-Kucera parameters; subsequently, it is identified using open-loop techniques. The controller used for closing the loop defines a subset of linear plants for which a Youla-Kucera parameter may be obtained. However, the method does not identify the parameters of the nonlinear system, but the parameters of a linearized model. In [6], the closed loop output error method, previously

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employed for discrete-time linear plants [3] is applied to nonlinear plants in closed loop with a nonlinear controller. A feature of this approach is the fact that the closed-loop system should be strongly strictly passive. This condition is too restrictive and can not be verified a priori in all cases since knowledge of the plant parameters would be necessary. It is worth mentioning that the aforementioned references do not present numerical simulations or experimental results. Further, these results would not be suitable for identifying certain class of nonlinear plants, for instance, nonlinear servomechanisms or robot manipulators. A depart from these previous methods is proposed in [7] where an inner-outer relay structure is proposed for identifying Coulomb friction in a positioning system, where the inner relay models the Coulomb friction. The outer relay is introduced to close the loop and generates periodic motion. Stability of the periodic solution of the closed-loop system is studied by means of the Poincaré map. Parameter estimation is performed through a recursive Least Squares algorithm. The method is tested experimentally showing good results; unfortunately, it does not provide parameter estimates of the linear part of the positioning system. The method proposed in [8] identifies a second order nonlinear system using a gradient algorithm. A high-gain controller stabilizes the open-loop unstable plant. That approach imposes differentiability conditions on the regressor vector and the paper only shows simulation.

This work outlines an approach for closed-loop identification of nonlinear servomechanism. This method extends previous results [9] to deal with a class of nonlinear servomechanisms. Moreover, compared with previous works [8], no differentiability conditions are imposed on the regressor vector. Experiments permit comparing the proposed approach against a standard Least Squares (LS) algorithm. These experimental results show that disturbances greatly affect the estimates obtained using the LS algorithm. In the case of the proposed approach, disturbances do not significantly influence the parameter estimates. The paper is organized as follows. Section 2 is devoted to the identification algorithm, stability properties and parameter convergence. Experimental results are shown in Section 3 and some concluding remarks are given in Section 4.

2 Closed-Loop Parameter Identification

2.1 Preliminaires

Fig. 1 depicts the idea of the closed loop identification algorithm. Two Proportional Derivative (PD) controllers simultaneously close the loop around the nonlinear servomechanism and its estimated model and an update law computes its parameter estimates. It is worth mentioning that both controllers have the same gains; moreover, under some mild conditions, the PD controller is able to stabilize the nonlinear servomechanism without knowledge of its parameters. Consider the nonlinear servomechanism model described by

$$\ddot{x} = -\gamma^T \mathrm{f}\left(x, \dot{x}\right) + bu \tag{1}$$

where $\dot{x}$ denotes the time derivative of x, $\mathbf{x} = [x, \dot{x}]^T$ are the states of the nonlinear servo, which are available through sensors, $\gamma \in \mathrm{R}^m$ and $b \in \mathrm{R}^+$ are unknown constant parameters, u is the input control, and $\mathrm{f}(\mathrm{x}) = \mathrm{f}(x, \dot{x}) \in \mathrm{R}^m$ is a nonlinear known function.

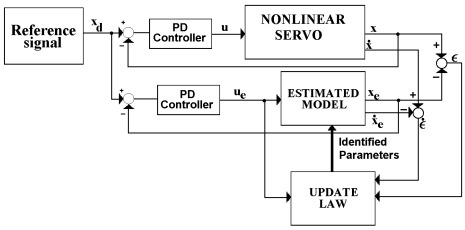

Fig. 1. Schematic for the proposed parameter identification algorithm

Model (1) fulfills the following assumption: **(A1)** The nonlinear function f fulfills $\|\mathrm{f}(x)\| \le k_f, \|\mathrm{f}(x_1) - \mathrm{f}(x_2)\| \le L_1 \|x_1 - x_2\|$.

2.2 Stability Analysis of the Closed-Loop System

Consider the PD control law $u(t)=k_p e - k_d dx/dt$, where, $k_p>0$, $k_d>0$ and the error e is defined as $e := x_d - x$. The reference x_d fulfills the following assumption: **(A2)** The reference x_d is bounded, i.e. $|x_d| \le \beta$. Then, substituting u into (1) yields

$$\ddot{x} = -\gamma^T \mathrm{f}\left(x, \dot{x}\right) + bk_p e - bk_d \dot{x} \tag{2}$$

Firstly, it is necessary to find conditions under which controller $u(t)$ stabilizes system (1). To this end, consider the following Lyapunov function candidate $V_1(\mathrm{x}) = 0.5\mathrm{x}^T \begin{bmatrix} b(k_p + \mu k_d) & \mu \\ \mu & 1 \end{bmatrix} \mathrm{x}$, where $\mu>0$. Function V_1 is positive definite if $\mu < bk_d$. Now, by taking the time derivative of V_1 along the trajectories of (2) and using $\gamma_1=\|\gamma\|$, bounds on f(x), equation of $e(t)$ and bound on x_d leads to $\dot{V}_1 \le -(bk_d - \mu)\dot{x}^2 - \mu bk_p x^2 + \mu(\gamma_1 k_f + bk_p\beta)\,|x| + (\gamma_1 k_f + bk_p\beta)\,|\dot{x}|$. Let the PD controller gains to be written as $k_p=\alpha L_p$ and $k_d=\alpha L_d$. Then, assuming that the inequality $\mu<0.5bk_d$ holds, and considering $z^T=[|x|, |dx/dt|]$ and the following definitions: $\mathrm{M} = diag\{\mu bL_p, 0.5bL_d\}, \mathrm{B}^T = [\mu(\gamma_1 k_f + bk_p\beta) \quad \gamma_1 k_f + bk_p\beta]$ allows writing the bound dV_1/dt as

$$\dot{V}_1 \le -\alpha \mathrm{z}^T \mathrm{M}\mathrm{z} + \mathrm{B}^T \mathrm{z} \le -\alpha\lambda_{\min}(\mathrm{M})\,\|\mathrm{z}\|\,\left[\|\mathrm{z}\| - \|\mathrm{B}\|(\alpha\lambda_{\min}(\mathrm{M}))^{-1}\right] \tag{3}$$

Consider the set Ω defined as $\Omega := \{z : \|z\| < \alpha \|\mathrm{B}\| (\lambda_{min}(\mathrm{M}))^{-1}\}$. Then, dV_1/dt is negative definite if $\mathrm{z} \notin \Omega$. Therefore, the feedback system (2) is Uniformly Ultimate Bounded

(UUB) [10] and the region Ω can be made arbitrarily small if α increases, i.e., with a high gain controller.

2.3 Stability Analysis of the Error Dynamics

Consider the following model of the nonlinear servomechanism (1): $\ddot{x}_e = -\hat{\gamma}^T \mathrm{f}\left(x_e, \dot{x}_e\right) + \hat{b}u_e$, where $\mathbf{x}_e=[x_e, \dot{x}_e]^T$ is the estimated state and parameters $\hat{\gamma}$, $\hat{b}$ are the estimates of γ, b, respectively. Let the PD control law: $u_e = k_p e_e - k_d \dot{x}_e$, with $e_e:=x_d-x_e$. It is worth noting that the proportional and derivative gains for u and u_e are the same. Substituting u_e into the model of the nonlinear servomechanism leads to $\ddot{x}_e = -\hat{\gamma}^T \mathrm{f}\left(x_e, \dot{x}_e\right) + \hat{b}k_p e_e - \hat{b}k_d \dot{x}_e$. Define the error between the output of the nonlinear system (2) and its model as $\varepsilon:=x-x_e$. Therefore, the associated error dynamics corresponds to the following differential equation

$$\ddot{\varepsilon} = -bk_d \dot{\varepsilon} - bk_p \varepsilon - \gamma^T \left(\mathrm{f}(\mathrm{x}) - \mathrm{f}(\mathrm{x}_e)\right) + \tilde{\theta}^T \phi \tag{4}$$

where $\tilde{\theta}^T := \left[\hat{\gamma} - \gamma, \hat{b} - b\right], \phi := \left[\mathrm{f}(\mathrm{x}_e), -u_e\right]$. The stability analysis employs passivity arguments [12]. Define $\mathrm{E} := \mathbf{x}-\mathbf{x}_e = [\varepsilon, d\varepsilon/dt]^T$ and consider the storage function $H_1\left(\varepsilon, \dot{\varepsilon}\right) = 0.5\mathrm{E}^T \begin{bmatrix} bk_p & \mu \\ \mu & 1 \end{bmatrix} \mathrm{E}$. This function is positive definite if $\mu < \sqrt{bk_p}$. Taking the time derivative of H_1 along the trajectories of (4) leads to $\dot{H}_1 \le \tilde{\theta}^T \phi\left(\mu\varepsilon + \dot{\varepsilon}\right) - \frac{1}{2} bk_d \left(\mu\varepsilon + \dot{\varepsilon}\right)^2 - \frac{1}{2}\mathrm{E}^T \mathrm{N}\mathrm{E} - \gamma^T \left(\mathrm{f}(\mathrm{x}) - \mathrm{f}(\mathrm{x}_e)\right)\left(\mu\varepsilon + \dot{\varepsilon}\right)$, where $\mathrm{N}=diag\{2\mu bk_p-\mu^2 bk_d, bk_d-2\mu\}$. Matrix N is positive definite if $\mu < \min\{2k_p/k_d, bk_d/2\}$. Define $\Xi := [|\varepsilon|, |d\varepsilon/dt|]^T$ and consider the following upper bounds $\gamma^T \left(\mathrm{f}(\mathrm{x}) - \mathrm{f}(\mathrm{x}_e)\right) \le \gamma_1 L_1 \|\mathrm{E}\|$, $\mu\varepsilon + \dot{\varepsilon} \le \sqrt{\mu^2 + 1}\|\Xi\|$. Since $\|\mathrm{E}\| = \|\Xi\|$, it follows that $\gamma^T \left(\mathrm{f}(\mathrm{x}) - \mathrm{f}(\mathrm{x}_e)\right)\left(\mu\varepsilon + \dot{\varepsilon}\right) \le \gamma_1 L_1 \sqrt{\mu^2 + 1}\|\mathrm{E}\|^2$. Therefore, dH_1/dt is upperbounded as $\dot{H}_1 \le \tilde{\theta}^T \phi\left(\mu\varepsilon + \dot{\varepsilon}\right) - \frac{1}{2} bk_d \left(\mu\varepsilon + \dot{\varepsilon}\right)^2 - \left[\frac{1}{2}\lambda_{\min}(\mathrm{N}) - \gamma_1 L_1 \sqrt{\mu^2 + 1}\right]\|\mathrm{E}\|^2$ and if $\lambda_{\min}(\mathrm{N}) > 2\gamma_1 L_1 \sqrt{\mu^2 + 1} > 0$ holds, then $\dot{H}_1 \le \tilde{\theta}^T \phi\left(\mu\varepsilon + \dot{\varepsilon}\right) - \frac{1}{2} bk_d \left(\mu\varepsilon + \dot{\varepsilon}\right)^2$. The above means that (4) defines an output strictly passive (OSP) mapping described as $\tilde{\theta}^T \phi \to \left(\mu\varepsilon + \dot{\varepsilon}\right)$. As a consequence, the output defined as ($\mu\varepsilon + \dot{\varepsilon}$) belongs to the space L_2. Then, ε is the output of a linear exponentially stable system whose input

belongs to the space L_2, thus [13]: $\varepsilon \to 0$ as $t \to \infty$. The last analysis makes intuitive to consider the next parameter updating law

$$\dot{\tilde{\theta}} = -\Gamma\phi\left(\mu\varepsilon + \dot{\varepsilon}\right) \tag{5}$$

where $\Gamma>0$, $\Gamma = \Gamma^T \in \mathbb{R}^{2\times 2}$. Let the storage function $H_2(\tilde{\theta}) = 0.5\tilde{\theta}^T\Gamma^{-1}\tilde{\theta}$, whose time derivative along the trajectories of (5) leads to $\dot{H}_2(\tilde{\theta}) = \left(\mu\varepsilon + \dot{\varepsilon}\right)\left(-\tilde{\theta}^T\phi\right)$. Consequently, (5) defines a passive operator $\left(\mu\varepsilon + \dot{\varepsilon}\right) \to \left(-\tilde{\theta}^T\phi\right)$. Therefore, the interconnected system (4)-(5) defines an OSP operator with output ($\mu\varepsilon + \dot{\varepsilon}$) [12]. Let $H=H_1+H_2$; it is not difficult to conclude that $dH/dt \leq -(1/2)bk_d(\mu\varepsilon + \dot{\varepsilon})^2$ and as result $\{\varepsilon, d\varepsilon/dt, \tilde{\theta}\} \in L_\infty$. Moreover, it was previously shown that (2) is UUB; therefore, $\{x, dx/dt\} \in L_\infty$ and so do $\{x_e, dx_e/dt\}$. Based on the last analysis, it is possible to state the following result.

Proposition 1. Consider the feedback interconnected system (4)-(5), assume that parameter μ satisfies: $\mu < \min\{2k_p/k_d, bk_d/2, sqtr(bk_p)\}$ and that $\lambda_{\min}(\mathrm{N}) > 2\gamma_1 L_1\sqrt{\mu^2+1} > 0$ holds; then, $\{\varepsilon, d\varepsilon/dt, \tilde{\theta}, x_e, dx_e/dt\} \in \mathrm{L}_\infty$ and $\varepsilon \to 0$ as $t \to \infty$.

2.4 Parameter Convergence

The following persistence of excitation (PE) condition ensures parameter convergence [11].

Definition 1. *A vector* $\phi : R^+ \to R^{2n}$ *is persistently exciting if there exist* $\alpha_1>0$, $\alpha_2>0$ *and* $\delta>0$ *such that* $\alpha_1 I \leq \int_{t_0}^{t_0+\delta} \phi(\tau)\phi^T(\tau)d\tau \leq \alpha_2 I$.

3 Experimental Results

The following nonlinear rotating servomechanism model allows testing the proposed methodology

$$J\ddot{x} + f\dot{x} + gml\sin(x) = \tau \tag{6}$$

Angular position, velocity and acceleration correspond to x, dx/dt, d^2x/dt^2, respectively; J and f are the servomechanism inertia and viscous damping respectively; $gmlsin(x)$ is a gravitational torque due to a pendulum of length l and mass m; g is the gravity constant, and τ the torque provided by a DC motor. If the amplifier driving the motor works in current mode, it is reasonable to assume that

$\tau=ku$, i.e., the torque provided by the motor is proportional to the voltage control signal. Fig. 2 depicts the convention for measuring the angular position x. Rewriting model (6) yields $\ddot{x} = -\gamma^T \mathrm{f}(x) + bu$, where $f(x) = [sin(x), dx/dt]^T$, $\gamma = [gml/J, f/J]$, $b=k/J$. The servomechanism employed for the experiments (Fig. 2) is a DC brushed motor controlled by a Copley Controls power amplifier, model 413, configured in current mode. The angular position is measured using a BEI optical encoder with a resolution of 2500 pulses per revolution and it is directly coupled to the motor shaft. Data acquisition is performed by a MultiQ 3 card from Quanser Consulting endowed with inputs for optical encoders. The electronics associated to these inputs multiply by 4 the encoder resolution. Angular position is scaled down by a factor of 10,000 corresponding to one motor shaft turn. The card has 12 bits digital-analog converters with an output voltage of $\pm 5V$. Programming was performed using the Matlab-Simulink software operating with the WINCON software from Quanser Consulting. Sampling period was 1 *ms*. The experiments are divided into three parts. In the first part, parameters of the nonlinear servomechanism are identified using a standard Least Squares (LS) algorithm and the proposed methodology without adding external disturbances. In the second part, the parameter estimates obtained in the first part allows designing a tracking controller that is applied to the nonlinear servomechanism. Finally, the experiments previously performed in the first part are repeated under a constant disturbance applied to the nonlinear servomechanism.

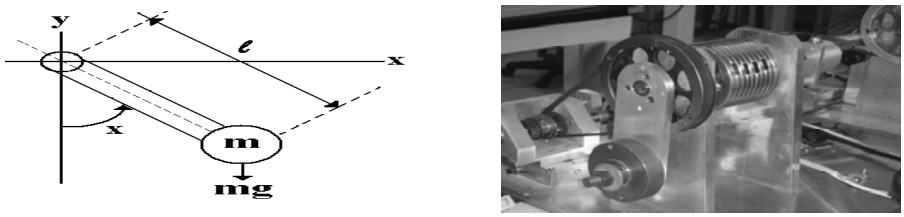

Fig. 2. Convention for measuring x (left hand side); laboratory prototype (right hand side)

3.1 Parameter Identification without Adding a Disturbance

The PD controller gains are set to $k_p=20$ and $k_d=1.3$. Velocity estimates dx_f/dt of dx/dt were obtained through filtering of the position measurements with $dx_f/dt=d\omega/dt$ and $d^2\omega/dt^2+1100\, d\omega/dt+30000\, \omega = 300000x$. The above filter is also employed in the model even if velocity dx_e/dt is available. The following relationships allow implementing the LS algorithm (see also Fig. 3)

$$\begin{aligned} &\ddot{y}_1 + \lambda_1 \dot{y}_1 + \lambda_2 y_1 = \lambda_2 \sin(x); \phi_1 = y_1; \ddot{y}_2 + \lambda_1 \dot{y}_2 + \lambda_2 y_2 = \lambda_2 \dot{x}; \phi_2 = \dot{y}_2 \\ &\ddot{y}_3 + \lambda_1 \dot{y}_3 + \lambda_2 y_3 = \lambda_2 u; \phi_3 = y_3; \ddot{y}_4 + \lambda_1 \dot{y}_4 + \lambda_2 y_4 = \lambda_2 x; z = \ddot{y}_4 \end{aligned} \tag{7}$$

with $\lambda_1=40$, $\lambda_2=400$, $z = \theta^T \phi_{LS}$ and $\phi_{LS} = [\phi_1 \quad \phi_2 \quad \phi_3]$. Note that equation for z holds for all time t; then it is valid for the instants $h, 2h, 3h, ..., (k-1)h, kh$, where h is the sampling period. Then, z can be rewritten as $z(kh)=\theta^T \phi(kh)$; omitting the sampling

period yields $z(k)=\theta^T \phi(k)$. The above expression allows using the Least Squares algorithm with forgetting factor [14]. The LS algorithm is implemented using $P(0)=diag\{500,500,500\}$ and a forgetting factor constant value of $\lambda=0.999$. The gains for the proposed algorithm were set to $\Gamma=diag\{500,15000,10000\}$ and $\mu=0.5$. The nonlinear servomechanism is excited using a signal provided by the Band-Limited White Noise block from MatLab Simulink (see Fig. 3) set to: noise power of 5, sample time of 0.1 and seed [12121]. Fig. 4 (a) and (b) show the time evolution of the parameter estimates obtained using the LS algorithm and the proposed approach respectively.

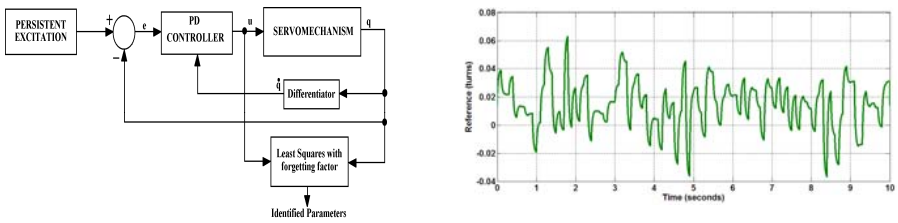

Fig. 3. Block diagram for implementing the Least Squares algorithm (left hand side); reference signal for the identification experiments (right hand side)

For the LS algorithm the parameters estimates approximately settle at $\{\hat{\gamma}^T, \hat{b}\}=\{28,8,2\}$ whereas for the proposed algorithm the corresponding values are $\{\hat{\gamma}^T, \hat{b}\}=\{26,10,3\}$. From the above it is clear that both algorithms give similar parameter estimates.

3.2 Trajectory Tracking Experiments

Let $\{\bar{\gamma}^T, \bar{b}\}=\{\bar{\gamma}_1, \bar{\gamma}_2, \bar{b}\}$ be the estimated parameters obtained with any of the parameter identification algorithms during the first ten seconds. Then, consider the control law $\bar{u}=(\bar{b})^{-1}\left[\bar{\gamma}^T \mathrm{f}\left(x,\dot{x}\right)+\ddot{x}_d+k_1\dot{e}+k_2 e\right]$, with k_1,k_2 being positive constants and e is the tracking error. The above control law applied to (1) leads to the following closed-loop error dynamics $\ddot{e}(t)+k_1\dot{e}+k_2 e=(\gamma-\bar{\gamma})^T \mathrm{f}\left(x,\dot{x}\right)+(\bar{b}-b)u$.

Note that if the parameter estimate errors $\gamma-\bar{\gamma}$ and $\bar{b}-b$ are zero, then, the tracking error converges to zero. In practice, good parameter estimate would lead to smaller tracking error e. The reference is a sinusoid with amplitude of *0.25* motor shaft turns and frequency of *4 rad/s*. Fig. 5 (a) and (b) show the tracking error when the control law $\bar{u}$ is computed using the LS and the proposed algorithm, respectively. The trajectory errors are similar.

3.3 Parameter Identification under Constant Disturbances

The parameter estimation algorithms are tested under the same conditions used previously, however, a constant disturbance *d=0.5 v* is considered in the model of the nonlinear servomechanism: $\ddot{x} = -\gamma^T \mathrm{f}(x) + bu + d$. Fig. 6 (a) and (b) depict the results for the LS and the proposed identification algorithm, respectively.

Note that the parameter estimates obtained using the LS algorithm are far different from the estimates obtained under no disturbance conditions. In particular, note that parameter $\hat{\gamma}_2$ takes negative values. On the other hand, the parameter estimates obtained with the proposed method stay near to the values obtained under no disturbance conditions.

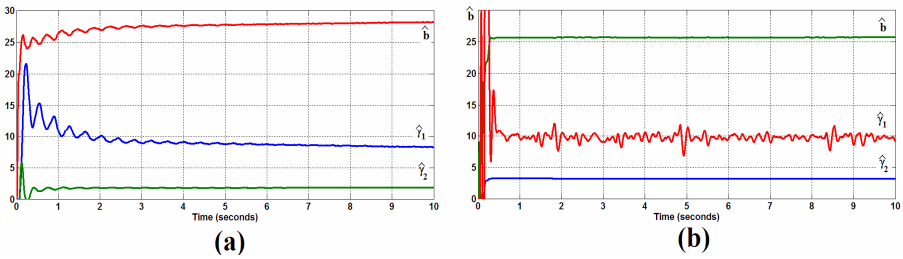

Fig. 4. Parameter estimates (non perturbed case): (a) LS algorithm; (b) proposed method

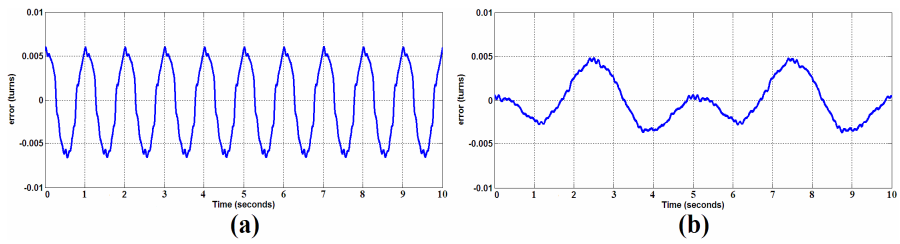

Fig. 5. Tracking error: (a) LS algorithm; (b) proposed algorithm

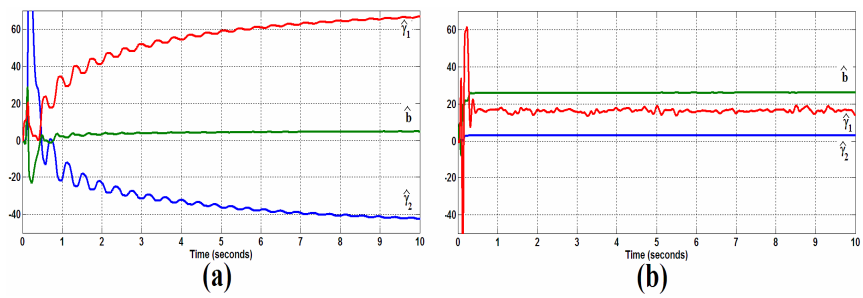

Fig. 6. Parameter estimates (perturbed case): (a) LS algorithm; (b) proposed method

4 Conclusion

This paper presents a new methodology for parameter identification of a class of nonlinear servomechanisms. The key element is a closed-loop identification technique where a Proportional Derivative controller stabilizes the servomechanism without knowledge of its parameters. The proposed identification algorithm and a standard Least Squares method are experimentally compared using a laboratory prototype. The experiments show that both algorithms give similar results when disturbances are not considered and produce low tracking errors when the parameter estimates are used for designing a trajectory-tracking algorithm; however, the proposed algorithm gives reasonable results in face of constant disturbances whereas the Least Squares method produces estimates far from the values obtained in the non-disturbance case.

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