AUTOTUNING OF A DC SERVOMECHANISM USING CLOSED LOOP IDENTIFICATION

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***Abstract*— In this paper a methodology for automatic tuning of position controlled servomotors is presented. The key element is a novel closed loop identification technique where the loop is closed using a standard Proportional Derivative controller. Once the servomotors parameters are estimated they are employed for setting up a controller using Linear Quadratic Regulator techniques. It is experimentally shown that within a few seconds the closed loop system is tuned.**

I. INTRODUCTION

In the early days of motion control selecting the parameters of a compensation filter for servomotor control was a time consuming and laborious task. Today, much of the task can be automated depending on what is known about the servo. For the user, it would be desirable to have an automatic tuning procedure allowing for a fast commissioning of the motion system. Automatic tuning or autotuning for short is composed of two sequential parts. In the first part parameters of the servo are identified. In the second a control law is computed using the parameters obtained in the first part. Regarding the second part there exists a great number of designs including linear and nonlinear techniques. Among the linear techniques Proportional Derivative (PD) and Proportional Integral Derivative (PID) controllers are well established in the realm of motion control. On the other hand, regarding the identification part situation is not as good. Even if there exist a lot of work concerning parameter identification [14], [15] most of the proposed algorithms deal with open loop stable systems. Note that for a linear model of position controlled servomotor its transfer function has a pole on the imaginary axis then making the open loop system marginally stable, then, most of the identification methods do not apply for this case. Moreover, if the identification is performed when the servomotor is coupled to a mechanical load, for example to a robot arm, closed loop identification with the loop closed around a position sensor would be desirable for security reasons since open loop techniques would lead to unbounded motor behavior. From the above situation it would be desirable to develop algorithms for fast identification and tuning of position controlled servos. In the following paragraphs a brief literature review concerning identification and autotunig is given.

There are several works dealing with parameter estimation of servomotors and among them two classes may be distinguished: open loop based and closed loop based. In the open loop case it is assumed than the servomotor works in velocity mode, i.e., the variable of interest is the servomotor velocity. An advantage of this approach is the fact that in this case the servomotor is open loop stable and then a standard off-line or on-line identification technique may be applied. Works in this vein are [1], [2], [3], [4], [5], [6] and [7]. It is interesting to mention that in [6] and [7] closed loop identification techniques are reported for velocity controlled servos, in particular, in [6] a Proportional Integral controller (PI) is employed for identifying a discrete-time model of a three-mass electromechanical system. Closed loop identification of position controlled servos is studied in [9], [10], [11]. In all of these works a technique used to obtain a stable movement is to close the feedback loop using a relay in the same way as in industrial processes [13] for controller tuning purposes. A drawback of the relay technique is the fact that tuning of the relay controller can be cumbersome, further, it is not clear that the signal generated by the relay have an adequate frequency spectrum to identify the servomotor parameters, specially if the parameters of non linear terms such that nonlinear friction and nonlinear position depending loads need to be identified. Finally, works presenting identification and tuning results simultaneously are [8] using relay techniques and [12] where a robust controller is designed from results using classic identification algorithms.

In this work a new automatic tuning methodology is presented. The loop around the servo is closed using a PD controller, in this way closed loop stability is easily ensured. The key ingredient of this methodology is a novel closed loop identification technique. Once the servomotor parameters are identified, a PD controller is computed using Linear Quadratic Regulator (LQR) techniques. It is experimentally shown that after several seconds the servo motor is tuned. Other controller techniques may be accommodated under this approach, moreover, the methodology may be applied to DC brush and AC brushless servos provided that an inner current loop is closed, i.e., the servoamplifier feeding the servomotors is working in current mode.

The paper is organized as follows. Section 2 is devoted to the identification algorithm. Section 3 deals with the LQR controller design. Experimental results for identification and tuning are shown in Section 4 and some concluding remarks are given in Section 5.

II. Closed Loop Parameter Identification

The idea of the closed loop identification algorithm is as follows (see Fig.1). Two PD controllers are applied to the real servomotor and to a servomotor model. Note that the same controller gains are used in both controllers. A reference consisting of a signal fulfilling the persistence of excitation condition [16] is fed to both closed loop systems. The error between the output of these systems and its time derivative feeds an identification algorithm and the model parameters are adjusted using the parameters obtained from the identification algorithm. It is clear that even if the closed loop system associated to the real servo is stable, the same conclusion can not be stated for the closed loop corresponding to the identified model then making necessary to analyze the stability of the later.

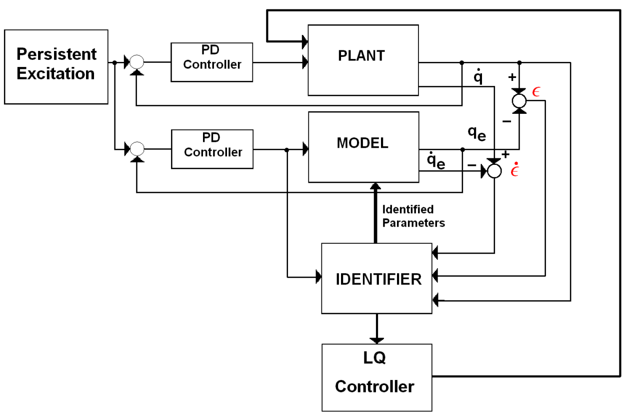


Figure 1. Blocks diagram of the autotuning process

The mathematical description of the DC servomechanism is given by

$$J\ddot{q} + f\dot{q} = \tau = ku \tag{1}$$

Where J and f are the servo inertia and viscous friction respectively, $\tau = ku$ the input torque, u the control input voltage and k the amplifier gain. Model (1) assumes that the servo electric time constant is small. The above is reasonably for small servomotors; however, in large servos this condition may be not fulfilled. However, by closing a current loop around the amplifier feeding the servomotor, a common industrial practice, allows to effectively reduce the electric time constant. We have from (1)

$$\ddot{q} = -a\dot{q} + bu \tag{2}$$

where $a = f/J, b = k/J$. Let the PD control law

$$u = k_p e - k_d \dot{q} \tag{3}$$

where

$$e = q_d - q \tag{4}$$

is the position error and q_d a reference. Consider now the following estimated model, with $\hat{a}$ and $\hat{b}$ being estimates of a and b

$$\ddot{q}_e = -\hat{a}\dot{q}_e + \hat{b}u_e \tag{5}$$

to which is applied a PD control law

$$u_e = k_p e_e - k_d \dot{q}_e \tag{6}$$

with

$$e_e = q_d - q_e \tag{7}$$

Note that the same gains are used in (3) and (6). Substituying (3) into (2) and (6) into (5) we have

$$\ddot{q} = -a\dot{q} + bk_p e - bk_d \dot{q} \tag{8}$$

$$\ddot{q}_e = -\hat{a}\dot{q}_e + \hat{b}k_p e_e - \hat{b}k_d \dot{q}_e \tag{9}$$

Let define the error between the outputs of the plant and the model

$$\epsilon = q - q_e \tag{10}$$

whose second time derivative is

$$\begin{aligned} \ddot{\epsilon} &= \ddot{q} - \ddot{q}_e \\ &= \left(-a - bk_d\right)\dot{\epsilon} - bk_p\epsilon + \left(\hat{a} - a\right)\dot{q}_e \\ &\quad + \left(\hat{b} - b\right)\left[k_d\dot{q}_e - k_p e_e\right] \end{aligned} \tag{11}$$

where (8) and (9) were employed. Let define

$$c = a + bk_d \tag{12}$$

where $c > 0$ and consider the parameter vector $\tilde{\theta}$ and vector ϕ given by

$$\tilde{\theta} = \hat{\theta} - \theta = \begin{bmatrix} \hat{a} - a \\ \hat{b} - b \end{bmatrix} \tag{13}$$

$$\phi = \begin{bmatrix} \dot{q}_e \\ k_d\dot{q}_e - k_p e_e \end{bmatrix} \tag{14}$$

then, (11) can be writen as

$$\ddot{\epsilon} = -c\dot{\epsilon} - bk_p\epsilon + \tilde{\theta}^T\phi \tag{15}$$

A. Stability

In the following stability analysis for differential equation (15) we propose the Lyapunov function candidate

$$V\left(\epsilon, \dot{\epsilon}, \tilde{\theta}\right) = \frac{1}{2}\dot{\epsilon}^2 + \frac{1}{2}bk_p\epsilon^2 + \mu\epsilon\dot{\epsilon} + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{16}$$

We first verify that (16) is positive defined, so we rewrite it as

$$V\left(\epsilon, \dot{\epsilon}, \tilde{\theta}\right) = \frac{1}{2}\left[\left(\dot{\epsilon} + \mu\epsilon\right)^2 + \left(bk_p - \mu^2\right)\epsilon^2\right] + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{17}$$

so, in order to have $V > 0$ the following inequality should be fulfilled

$$0 < \mu \leq \sqrt{bk_p} \tag{18}$$

and under condition (18) we take the time derivate of (16)

$$\begin{aligned} \dot{V}\left(\epsilon, \dot{\epsilon}, \tilde{\theta}\right) &= \dot{\epsilon}\ddot{\epsilon} + bk_p\epsilon\dot{\epsilon} + \mu\left[\epsilon\ddot{\epsilon} + \dot{\epsilon}^2\right] + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \quad (19) \\ &= -(c-\mu)\dot{\epsilon}^2 - \mu bk_p\epsilon^2 - \mu c\epsilon\dot{\epsilon} + \tilde{\theta}^T\phi\dot{\epsilon} \\ &\quad +\tilde{\theta}^T\mu\phi\epsilon + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \end{aligned}$$

let define $\alpha = c - \mu > 0$, then (19) can be writen as

$$\dot{V} = -\alpha\dot{\epsilon}^2 - \mu bk_p\epsilon^2 - \mu c\epsilon\dot{\epsilon} + \tilde{\theta}^T\left[\phi\mu\epsilon + \phi\dot{\epsilon} + \Gamma^{-1}\dot{\tilde{\theta}}\right] \tag{20}$$

from this last equation it is clear that the value of $\dot{\tilde{\theta}}$ must be updated as

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = -\Gamma\left(\phi\mu\epsilon + \phi\dot{\epsilon}\right) \tag{21}$$

then, (20) becomes

$$\dot{V} = -\alpha\dot{\epsilon}^2 - \mu bk_p\epsilon^2 - \mu c\epsilon\dot{\epsilon} \tag{22}$$

Completing squares the last equation can be written as

$$\dot{V} = -\alpha\left[\left(\dot{\epsilon} + \frac{\mu c\epsilon}{2\alpha}\right)^2 + \left(\frac{\mu bk_p}{\alpha} - \frac{\mu^2c^2}{\alpha^2}\right)\epsilon^2\right] \tag{23}$$

A condition for negative definiteness of $\dot{V}$ is

$$\beta = \frac{\mu bk_p}{\alpha} - \frac{\mu^2c^2}{\alpha^2} > 0 \tag{24}$$

which is equivalent to the following inequality

$$\mu \leq \frac{4bk_pc}{4bk_p + c^2} \tag{25}$$

where the inequality $\alpha = c - \mu$ was used. From the above, boundedness of ϵ, $\dot{\epsilon}$ and $\tilde{\theta}$ is concluded. To show that ϵ and $\dot{\epsilon}$ converge to zero Barbalat lemma [16] is employed. To this end note from (23) that

$$\dot{V} \leq -\alpha\beta\epsilon^2 \tag{26}$$

integrating the above inequality yields

$$V - V(0) \leq -\int_0^t \alpha\beta\epsilon^2 d\rho \tag{27}$$

from which it can be shown that

$$\int_0^t \epsilon^2 \leq \frac{V(0)}{\alpha\beta} < \infty \tag{28}$$

and then ϵ belongs to the space L^2. From the above and since it was proved that ϵ and $\dot{\epsilon}$ are bounded it follows that ϵ and $\dot{\epsilon}$ converge to zero.

According to the preceding analysis, constant μ must fulfill the following inequality

$$\mu \leq \min\left[\frac{4bk_pc}{4bk_p + c^2}, \sqrt{bk_p}\right] \tag{29}$$

Finally, boundedness of the model output q_e is concluded from boundedness of ϵ and q.

Parameter convergence is obtained if the following persistence of excitation condition is fulfilled [16]

Definition 1: A vector $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{2n}$ is persistently exciting (PE) if there exist $\alpha_1, \alpha_2, \delta > 0$ such that

$$\alpha_2 I \geq \int_{t_0}^{t_0+\delta} \phi(\tau)\,\phi^T(\tau)\,d\tau \geq \alpha_1 I \tag{30}$$

for all $t_0 \geq 0$.

III. LQR CONTROLLER DESIGN

Because the parameter identification is performed on-line it is possible to carry out on-line the design of a LQR controller for the DC servomotor. To this end system (1) can be represented in space variables as

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & -a \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u \quad (31) \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned}$$

where $x_1 = q, x_2 = \dot{q}$ and $y = q$. Let define

$$\begin{aligned} e_1(t) &= q(t) - q_d \quad (32) \\ e_2(t) &= \dot{e}_1(t) = \dot{q}(t) \\ e_3(t) &= \int_0^t e_1(\tau)d\tau \end{aligned}$$

then we get the next state space representation

$$\begin{aligned} \dot{e}(t) &= \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -a & 0 \\ 1 & 0 & 0 \end{bmatrix} e + \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} u \quad (33) \\ y(t) &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} e \end{aligned}$$

The quadratic performance index is given by

$$\begin{aligned} J &= \frac{1}{2}e^T(t_f)Q_Te(t_f) \quad (34) \\ &\quad +\frac{1}{2}\int_{t_0}^{t_f}\left[e^T(t)Q_1(t)e(t) + u^T(t)Q_2(t)u(t)\right]dt \\ &= \frac{1}{2}\int_0^T\left\{e^T\begin{bmatrix} \gamma & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \eta \end{bmatrix}e + \kappa u^Tu\right\}dt \end{aligned}$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & -a & 0 \\ 1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, \\ Q_1 &= \begin{bmatrix} \gamma & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \eta \end{bmatrix}, Q_2 = \kappa, Q_T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

and γ, δ, η are used to weight the value of the states e_1, e_2, e_3 respectively, while κ is used to weight the magnitude of the control signal u. Let the matrix S

$$S(t) = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \tag{35}$$

Then, the optimal control law is given by

$$\begin{aligned} u^*(t) &= -Q_2^{-1} B^T(t) S(t) e(t) \\ &= -Le(t) \end{aligned} \tag{36}$$

where

$$\begin{aligned} L &= Q_2^{-1} B^T(t) S(t) \\ &= \frac{1}{\kappa} \begin{bmatrix} 0 & b & 0 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \\ &= \frac{\kappa}{q} \begin{bmatrix} S_{12} & S_{22} & S_{23} \end{bmatrix} \end{aligned} \tag{37}$$

The corresponding Riccati differential equation is given by

$$-\frac{dS(t)}{dt} = \dot{S} = Q_1 + A^T S + SA - SBQ_2^{-1} B^T S \tag{38}$$

The steady state solution $\dot{S} = 0$ is considered, then

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 \\ 0 & \delta & 0 \\ 0 & 0 & \eta \end{bmatrix} + \tag{39}$$

$$\begin{bmatrix} S_{13} & S_{23} & S_{33} \\ S_{11} - aS_{12} & S_{12} - aS_{22} & S_{13} - aS_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} S_{13} & S_{11} - aS_{12} & 0 \\ S_{23} & S_{12} - aS_{22} & 0 \\ S_{33} & S_{13} - aS_{23} & 0 \end{bmatrix}$$

$$-\frac{b^2}{\kappa} \begin{bmatrix} S_{12}^2 & S_{12}S_{22} & S_{12}S_{23} \\ S_{12}S_{22} & S_{22}^2 & S_{22}S_{23} \\ S_{12}S_{23} & S_{22}S_{23} & S_{23}^2 \end{bmatrix}$$

which yields

$$\begin{aligned} S_{11} &= -S_{23} + aS_{12} + \frac{b^2}{\kappa} S_{12} S_{22} \\ S_{12} &= \sqrt{\frac{\kappa}{b^2} (\gamma + 2S_{13})} \\ S_{13} &= aS_{23} + \frac{b^2}{\kappa} S_{22} S_{23} \\ S_{22} &= \frac{-2a\kappa + \sqrt{4a^2\kappa^2 + 4b^2\kappa(\delta + 2S_{12})}}{2b^2} \\ S_{23} &= \frac{\sqrt{\eta\kappa}}{b} \\ S_{33} &= \frac{b^2}{\kappa} S_{12} S_{23} \end{aligned} \tag{40}$$

Then, the corresponding control law is given as follows

$$\begin{aligned} u &= -Le(t) \\ &= -\frac{b}{\kappa} \begin{bmatrix} S_{12} & S_{22} & S_{23} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \\ &= -\frac{b}{\kappa} \left[e_1 S_{12} + e_2 S_{22} + e_3 S_{23} \right] \end{aligned} \tag{41}$$

IV. Experimental Results

The servomechanism employed for the experiments (Figure 2) is controlled by a Copley Controls power amplifier, model 413, configured in current mode. Angular position of the motor is measured by a BEI optical encoder. Resolution of the optical encoder is 2500 pulses per revolution and is directly coupled to the motor shaft.

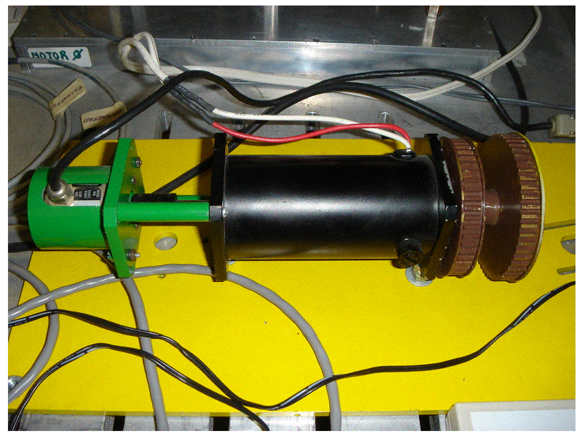


Figure 2. DC motor used in the laboratory test

Data adquisition is performed by a MultiQ 3 card from Quanser Consulting endowed with inputs for optical encoders. The electronics associated to these inputs multiply by 4 the encoder resolution. Angular position is scaled down by a factor of 10000 corresponding to one motor shaft turn. The card has 12 bits for digital-analog conversion with a output voltage range of $\pm 5V$ (volts).

Programming was performed using the Matlab-Simulink software operating with the WINCON software from Quanser Consulting. Sampling period was $1\ ms$. Figure 3 shows the platform used for experiments.

Figure 3. Experimental platform

Velocity estimates were obtained from position measurements through a high-pass filter. As it was indicate before, parameter identification and controller tuning are perfomed sequentially. In the first part of the experiment the loop is closed using a PD controller. It corresponds to the first 5s of the experiment where the values $\Gamma = 500$ and $\mu = 50$ were used for the identification algorithm. In the second part the LQR controller is computed using the estimates obtained from the first part using the values $\gamma = 1.5, \delta = 0.015, \eta = 0.001, \kappa = 0.06$.

Figure 4 shows the reference signal employed for the experiments. This signal is composed of two parts. The first part was generated using the block Band Limited White Noise and it is used for parameter identification during the first 5s. The second part is a square wave with an amplitude of 1 motor turn and a period of 5s.

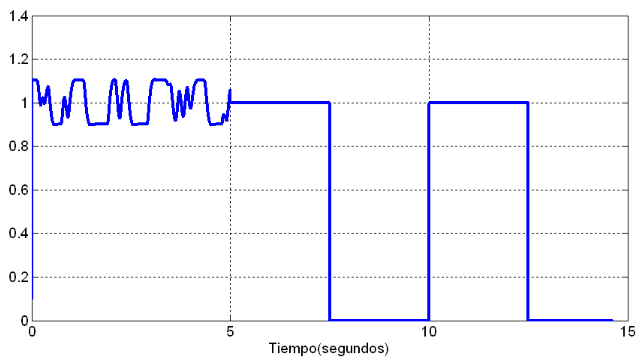


Figure 4. Reference signal q_d

The identified parameters are shown in the Figures 5 and 6, where the average values are $\hat{a} = 0.2, \hat{b} = 120$. Note that the servo parameters are identified within the first 5 seconds.

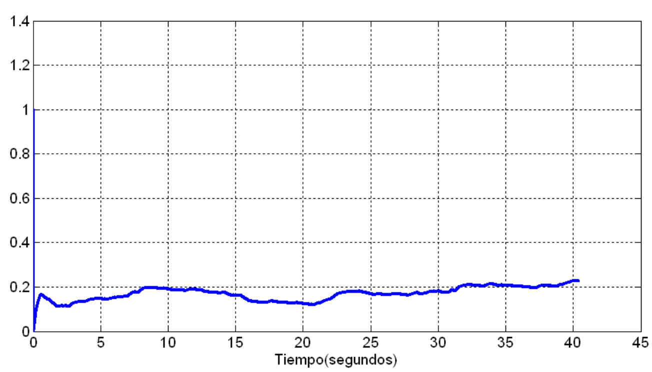


Figure 5. Parameter estimated $\hat{a}$

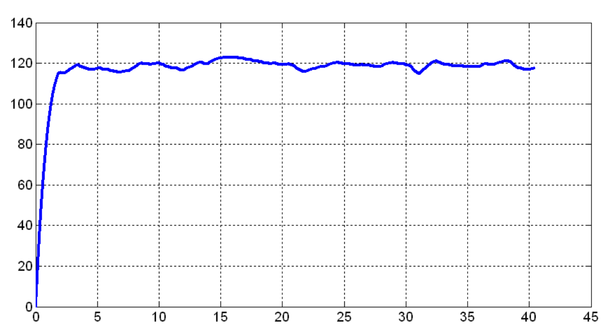
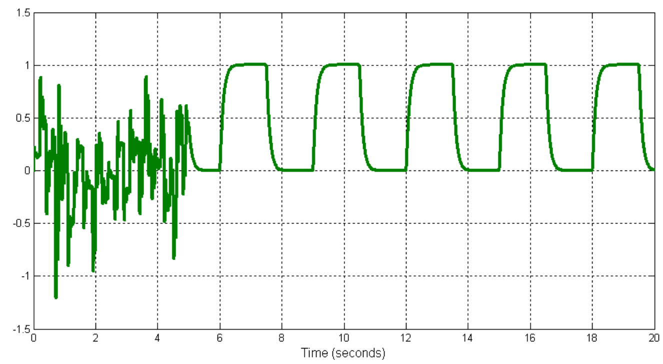
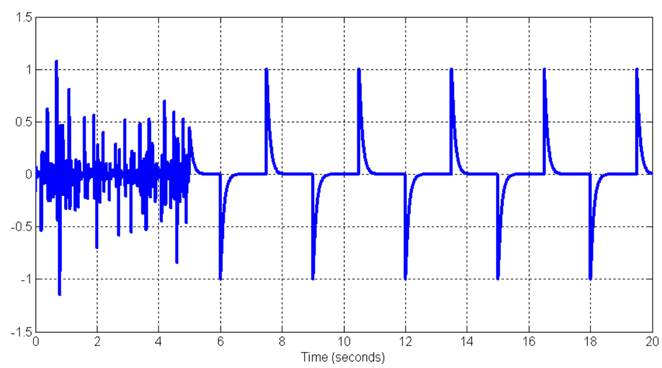


Figure 6. Parameter estimated $\hat{b}$

Figures 7 and 8 show the response of the servomotor. The first 5 seconds correspond to the servo response to the persisting exciting reference. The next seconds correspond to the servo response using the LQR controller.



Figue 7. Response of the system with closed loop identification and autotuning



Figue 8. Position error

V. CONCLUSION

In this paper a methodology for automatic tuning of servomotors is presented. The proposed approach is not based on relay techniques and its key feature is a novel closed loop identification technique where the loop is closed under PD control. Once the servo parameters are identified an LQR control law is computed then applied to the servomotor. It is experimentally shown that within a few seconds the servo is tuned. Future work includes to dispense velocity measurements in the identification algorithm and to use more elaborated controllers.

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