# CLOSED LOOP IDENTIFICATION OF A DC SERVOMECHANISM

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***Abstract*— A new method for closed loop identification of position controlled DC servomechanisms is proposed. The loop around the servo is closed using a Proportional Derivative (PD) controller. A model of the servo is simultaneously controlled using a second PD controller. The error and its time derivative between the output of both, the real servo and its model, is employed for identifying the motor parameters which in turn are employed for updating the model. Properties of the identification scheme are studied using Lyapunov stability theory. Experimental results on a laboratory prototype are shown to validate the proposed approach.**

## I. Introduction

Servomechanism parameter estimation is needed for fault detection and estimation and for controller tuning. Estimation may be performed in open loop or closed loop. In the first case there are several surveys reporting identification methodologies when it is assumed that the servo velocity and its armature current are available. In [1] is presented the parameter estimation of a direct current (DC) motor assuming linear models and using step signals. The estimation is performed in open loop using the properties of the step response of DC motor linear models. In [2] the off-line Least Squares algorithm in the frequency domain is applied. This algorithm is compared against a standard time domain Least Squares algorithm showing that the first has a better performance. An interesting feature is that the excitation signal used to perform the parameter estimation, however, the authors underline that the frequency approach is not suitable for on line DC motor parameter identification.

A interesting survey about nonlinear models estimation for DC motors is shown in [3]. In this case, using both velocity and armature current measurements and assuming that the motor works in open loop, the authors show that it is possible to estimate the nonlinear relationship between both the current and the voltage in the motor brushes. The estimation algorithm, which is off-line, is based in the minimization of a quadratic criterion performed using the Gauss-Newton algorithm. In this case the Hartley functions are used to convert a nonlinear differential equation into an algebraic relation in the same way that the Laplace transform is applied to a linear differential equation with constant parameters. The resulting algebraic expression is linear in the parameters, and they are estimated using an off-line Least Squares algorithm. The output variable is the armature current and the motor works in open loop.

The Least Squares Method is also employed in [4] where the output variable is the motor velocity and all the experiments are performed in open loop. In [5] the Maximum Likelihood method is proposed for servomotor identification considering the presence of nonlinear friction. A feature of the method is the use of multifrecuencial binary signals to excitate the system and then avoiding the identification errors introduced by the presence of Coulomb friction. The experiments were performed in open loop and the measurement variables are both angular and rectilinear velocities of the table driven by the motor through a ball screw.

In all the works mentioned above the identification experiments were performed in open loop. However, there are surveys [6], [7] reporting closed loop identification experiments. In [6] are presented both open and closed loop experiments applied to an electromechanical system made up of three masses impulsed by a DC motor. Identification was performed on line with a Recursive Least Square algorithm employing a discrete time model of the system. In the case of the closed loop experiments, the loop was closed using a Proportional-Integral controller (PI), taking as the controlled variable the motor velocity. A disadvantage of the method is the fact that in a discret model its parameters does not correspond to the physical system parameters, however, this issue is not important if the identified parameters are used for the design of discrete time linear controllers. In [7] is presented a nonlinear closed loop identification method to estimate nonlinearities when the motor performs reverse changes.

If the measured variable is the servomotor position, it is required to close the feedback loop because in this case the servomotor has a pole on the imaginary axis, and for this reason it is not possible to ensure stable operation in open loop. This fact is important and critic if the motor is connected to any dispositive or mechanical load as a robot or tool.

A technique used to obtain a stable movement is to close the feedback loop with a relay [8], [9], [10], [11] in the same way as in industrial processes [13] for controller tuning purposes. The stable oscillation allows to excitate the motor and then identify its parameters. This methodology is used in [9] and

[10] for the identification of Coulomb friction. An aspect that is not studied in these works is related to the spectral richness of the excitation signal applied to the system, wich in this case is the relay output.

No one of the above works shows explicity that the generated signal by the relay have a frequencial spectrum suitable for identification purposes, moreover, if the number of parameters to identify increases it is not proved that the signals generated in closed loop have the spectral richness necessary to identify the parameters.

Another approach not relying on relays is shown in [12] where the loop is closed using a Proportional Control law. In this case an off-line Least Square algorithm is used and the parameters obtained were used for the design of a robust controller.

In this work a novel method for closed loop identification of position controlled DC servomechanisms is proposed. The approach does not rely on relay feedback and a Proportional Derivative control algorithm is employed for closing the loop. In this way, the spectrum of the signals applied for identification purposes can be modified without any regard of the control law. Stability of the identification algorithm is studied using Lyapunov stability theory. The paper is organized as follows. Section 2 is devoted to the novel identification algorithm and its stability properties. Experimental results are shown in Section 3 and some concluding remarks are given in Section 4.

## II. PROBLEM STATEMENT

The idea behind the proposed identification algorithm is as follows (see Fig. 1). The DC servo is controlled using a PD control law with the servo position as output. In this way a stable behavior is obtained. A linear model of the servo is also controlled using a second PD algorithm. It is worth mentioning that both controllers use the same gains. The error between the servo output and its model feeds an identification algorithm. As it will be shown, the time derivative of this error is also used for identification purposes but it is not shown in Fig. 1 for the sake of simplicity. Parameter estimates obtained from the identification algorithm are then used to update the model. Even if the real servo has a stable behavior, it is not necessarily the case of the model, then, stability of the identified model as well as boundedness of the identified parameters becomes an issue.

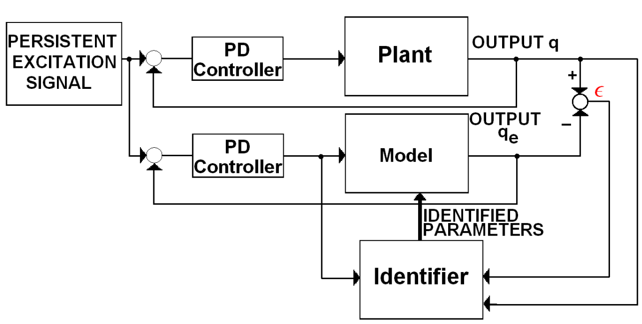



**Figure 1.** Blocks diagram of the closed loop identification process

The mathematical description of the CD servomechanism is given by

$$J\ddot{q} + f\dot{q} = \tau = ku \tag{1}$$

Where $J$ and $f$ are the servo inertia and friction respectively, $\tau = ku$ the input torque, $u$ the control input voltage and $k$ the amplifier gain. We have from (1)

$$\begin{aligned} \ddot{q} &= -\frac{f}{J}\dot{q} + \frac{k}{J}u \\ &= -a\dot{q} + bu \end{aligned} \tag{2}$$

where $a = f/J, b = k/J$. Let the PD control law

$$u = k_p e - k_d \dot{q} \tag{3}$$

where

$$e = q_d - q \tag{4}$$

is the position error and $q_d$ a reference. Consider now the estimated model, with $\hat{a}$ and $\hat{b}$ being estimates of $a$ and $b$

$$\ddot{q}_e = -\hat{a}\dot{q}_e + \hat{b}u_e \tag{5}$$

to wich is applied a PD control law

$$u_e = k_p e_e - k_d \dot{q}_e \tag{6}$$

with

$$e_e = q_d - q_e \tag{7}$$

Note that the same gains are used in (3) and (6). Substituying (3) into (2) and (6) into (5) we have

$$\ddot{q} = -a\dot{q} + bk_p e - bk_d \dot{q} \tag{8}$$

$$\ddot{q}_e = -\hat{a}\dot{q}_e + \hat{b}k_p e_e - \hat{b}k_d \dot{q}_e \tag{9}$$

Let define the error between the outputs of the plant and the model

$$\epsilon = q - q_e \tag{10}$$

whose second time derivative is

$$\begin{aligned} \ddot{\epsilon} &= \ddot{q} - \ddot{q}_e \\ &= \left(-a - bk_d\right)\dot{\epsilon} - bk_p\epsilon + \left(\hat{a} - a\right)\dot{q}_e \\ &\quad + \left(\hat{b} - b\right)\left[k_d\dot{q}_e - k_p e_e\right] \end{aligned} \tag{11}$$

where (8) and (9) were employed. Let define

$$c = a + bk_d \tag{12}$$

where $c > 0$ and consider the parameter vector $\tilde{\theta}$ and vector $\phi$ given by

$$\tilde{\theta} = \hat{\theta} - \theta = \begin{bmatrix} \hat{a} - a \\ \hat{b} - b \end{bmatrix} \tag{13}$$

$$\phi = \begin{bmatrix} \dot{q}_e \\ k_d \dot{q}_e - k_p e_e \end{bmatrix} \tag{14}$$

then, (11) can be writen as

$$\ddot{\epsilon} = -c\dot{\epsilon} - bk_p\epsilon + \tilde{\theta}^T \phi \tag{15}$$

### A. Stability

For following stability analysis we propose the Lyapunov function candidate

$$V\left(\epsilon, \dot{\epsilon}, \tilde{\theta}\right) = \frac{1}{2}\dot{\epsilon}^2 + \frac{1}{2}bk_p\epsilon^2 + \mu\epsilon\dot{\epsilon} + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{16}$$

First we verify that (16) is positive defined, so we rewrite it as

$$V\left(\epsilon, \dot{\epsilon}, \tilde{\theta}\right) = \frac{1}{2}\left[(\dot{\epsilon} + \mu\epsilon)^2 + \left(bk_p - \mu^2\right)\epsilon^2\right] + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} \tag{17}$$

so, in order to have $V > 0$ the following inequality should be fulfilled

$$0 < \mu \leq \sqrt{bk_p} \tag{18}$$

and under condition (18) we take the time derivate of (16)

$$\begin{aligned} \dot{V} &= \dot{\epsilon}\ddot{\epsilon} + bk_p\epsilon\dot{\epsilon} + \mu\left[\epsilon\ddot{\epsilon} + \dot{\epsilon}^2\right] + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \\ &= -(c-\mu)\dot{\epsilon}^2 - \mu bk_p\epsilon^2 - \mu c\epsilon\dot{\epsilon} + \tilde{\theta}^T\phi\dot{\epsilon} \\ &\quad +\tilde{\theta}^T\mu\phi\epsilon + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \end{aligned} \tag{19}$$

let define $\alpha = c - \mu > 0$, then (19) can be writen as

$$\dot{V} = -\alpha\dot{\epsilon}^2 - \mu bk_p\epsilon^2 - \mu c\epsilon\dot{\epsilon} + \tilde{\theta}^T\left[\phi\mu\epsilon + \phi\dot{\epsilon} + \Gamma^{-1}\dot{\tilde{\theta}}\right] \tag{20}$$

from this last equation it is clear that the value of $\dot{\tilde{\theta}}$ must be updated as

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = -\Gamma\left(\phi\mu\epsilon + \phi\dot{\epsilon}\right) \tag{21}$$

then, (20) becomes

$$\dot{V} = -\alpha\dot{\epsilon}^2 - \mu bk_p\epsilon^2 - \mu c\epsilon\dot{\epsilon} \tag{22}$$

Completing squares the last equation can be written as

$$\dot{V} = -\alpha\left[\left(\dot{\epsilon} + \frac{\mu c\epsilon}{2\alpha}\right)^2 + \left(\frac{\mu bk_p}{\alpha} - \frac{\mu^2c^2}{\alpha^2}\right)\epsilon^2\right] \tag{23}$$

A condition for negative definiteness of $\dot{V}$ is

$$\beta = \frac{\mu bk_p}{\alpha} - \frac{\mu^2c^2}{\alpha^2} > 0 \tag{24}$$

which is equivalent to the following inequality

$$\mu \leq \frac{4bk_pc}{4bk_p + c^2} \tag{25}$$

where the inequality $\alpha = c - \mu$ was used. From the above, boundedness of $\epsilon$, $\dot{\epsilon}$ and $\tilde{\theta}$ is concluded. To show that $\epsilon$ and $\dot{\epsilon}$ converge to zero Barbalat lemma [16] is employed. To this end note from (23) that

$$\dot{V} \leq -\alpha\beta\epsilon^2 \tag{26}$$

integrating the above inequality yields

$$V - V(0) \leq -\int_0^t \alpha\beta\epsilon^2 d\rho \tag{27}$$

from which it can be shown that

$$\int_0^t \epsilon^2 \leq \frac{V(0)}{\alpha\beta} < \infty \tag{28}$$

and then $\epsilon$ belongs to the space $L^2$. From the above and since it was proved that $\epsilon$ and $\dot{\epsilon}$ are bounded it follows that $\epsilon$ and $\dot{\epsilon}$ converge to zero.

Acording to the analysis, constant $\mu$ must fulfill the following inequality

$$\mu \leq \min\left[\frac{4bk_pc}{4bk_p + c^2}, \sqrt{bk_p}\right] \tag{29}$$

Finally, boundedness of the model output $q_e$ is concluded from boundedness of $\epsilon$ and $q$.

Parameter convergence is obtained if the following persistence of excitation condition is fulfilled [16]

*Definition 1:* A vector $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{2n}$ is persistently exciting (PE) if there exist $\alpha_1, \alpha_2, \delta > 0$ such that

$$\alpha_2 I \geq \int_{t_0}^{t_0+\delta} \phi(\tau)\phi^T(\tau)\,d\tau \geq \alpha_1 I \tag{30}$$

for all $t_0 \geq 0$.

## III. Experimental results

The servomechanism employed for the experiments (Figure 2), wich is controled by an Copley Controls power amplifier model 413 configured in current mode. Angular position of the motor is measured by an optical encoder. Resolution of the optical encoder is 2500 pulses per revolution and is directly coupled to the motor shaft.

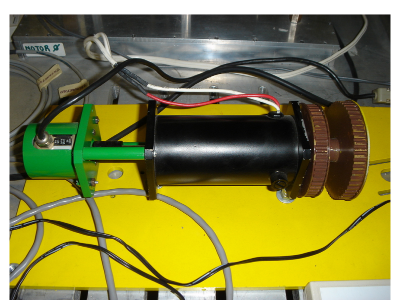


**Figure 2**. DC motor used in the laboratory test

Data adquisition is performed by a MultiQ 3 card from Quanser Consulting with inputs for optical encoders. The electronics associated to these inputs multiply by 4 the resolution of the encoder. Angular position is scaled down by an factor of 10000 that correspond to one motor shaft turn. The card has 12 bits for digital-analogic conversion with a output voltage range of $\pm 5V$ (volts).

Programming was performed using the Matlab-Simulink software operating with the WINCON software from Quanser Consulting . The sampling period was 1 $ms$.

**Figure 3.** Experimental platform

Figure 3 shows the experimental platform used for the implementation of the identification algorithm. Identification experiments were performed using $\Gamma = 500$ and $\mu = 50$. Figure 4 shows the reference signal employed for the experiments. Such a signal was created using the block Band-Limited White Noise from the library of Matlab-Simulink.

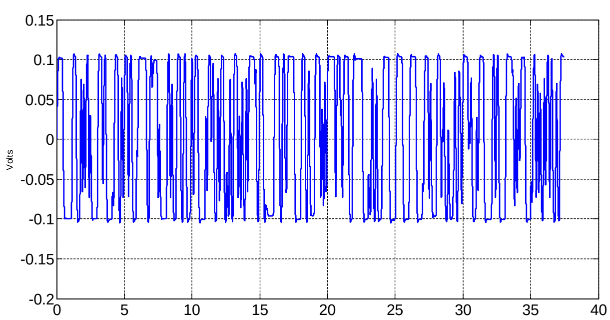



**Figure 4.** Reference signal $q_d$

The response of both servomechanism and model are shown in Figures 5 and 6 respectively.

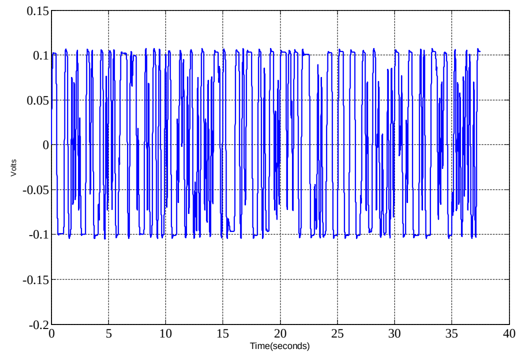



**Figure 5.** Signal $q$

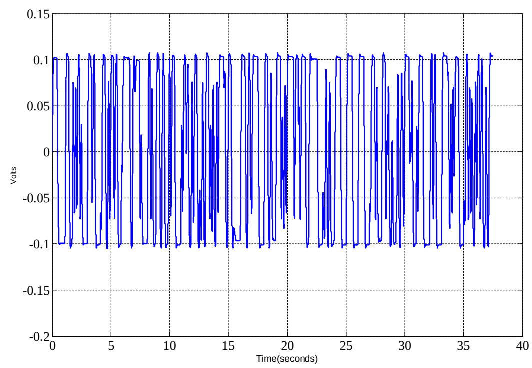



**Figure 6.** Signal $q_e$

The identified parameters are shown in the Figures 7 and 8, where the average values $\hat{a} = 0.2, \hat{b} = 120$ are obtained.

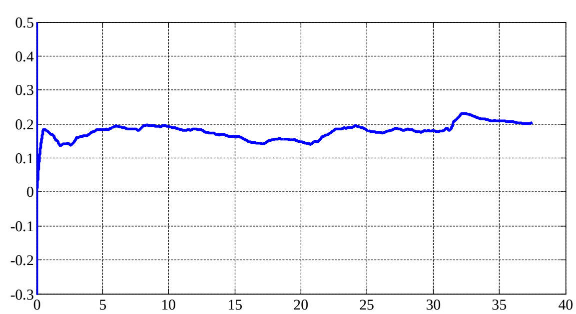



**Figure 7.** Parameter estimated $\hat{a}$

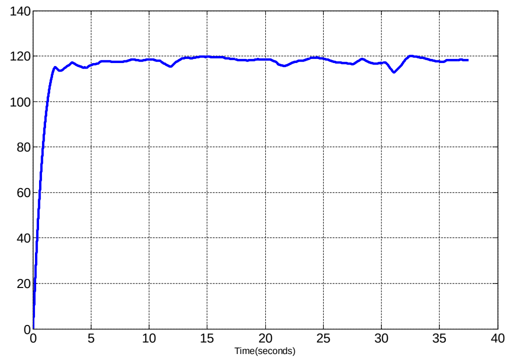
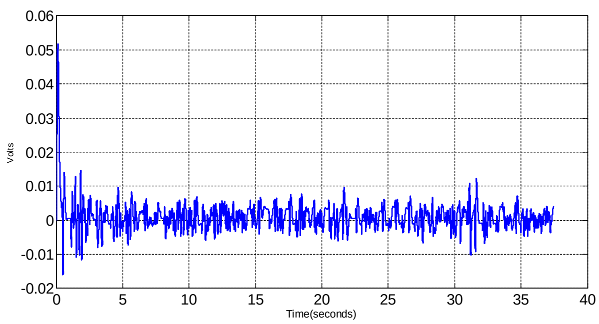



**Figure 8.** Parameter estimated $\hat{b}$

Finally figure 9 shows error $\epsilon$.




**Figue 9.** Error $\epsilon$

## IV. Conclusion

In this work a novel on-line parameter identification scheme applied to position controlled DC servomechanisms is presented. The method is applied in closed loop and it is not based on relay feedback. Properties of the identification algorithm are studied using Lyapunov stability theory and its performance is evaluated through experiments using a laboratory prototype. Future research is aimed towards the identification of servo nonlinear models containing nonlinearities such that Coulomb friction and position dependent nonlinearities. Another venue of research is the possibility of using the presented scheme for fault detection and for automatic controller tuning.

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